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ExP

Journal of
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VOL. IX., No. 5 October, 1926

THE FIXATIONAL PAUSE OF THE EYES

BY PERCY W. COBB AND FRANK K. MOSS

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INTRODUCTORY

It has been found that work done under the guidance of the eyes is facilitated, in some cases, by an increase in the amount of light. Investigations which have shown this are of two types. First, tests in factories under actual working conditions show an increased output of work under a higher intensity of illumination (1). Secondly, in the laboratory the same problem has been attacked in an analytical way. The threshold-time of exposure of a test-object has been found, in general, to be shorter with the higher illumination (2, 3). Beyond, say, 0.3 sec. the time of exposure is hardly critical. Under ordinary working illuminations, what is not seen within that time would seldom, if ever, be seen. The threshold exposure-time has been found to vary from about this value to a very few thousandths of a second, depending upon the conditions. This is one datum in the problem.

With the knowledge of the least time that an image must be allowed to act upon the retina to determine an appropriate response, the question at once arises: In the ordinary use of the eyes, what is the least time that the image is, in fact, allowed to act?

The eyes are continually in motion, and their movement has been shown to consist mainly of rapid sweeps, long or short, with brief pauses. During the sweeps it is obviously

impossible for the details of objects to make a sufficient impression, since at these times the image is rushing across the retina at a corresponding rate. It is only during the fixational pauses in the eye-movements, when the image is nearly or quite at rest upon the retina, that an adequate impression can be made. The duration of the shortest eye-pause therefore becomes of importance in conjunction with our knowledge of the threshold-time. Unless the pause of the eyes is long enough for the image, such as it is, to make a sufficient impression upon the retina, there can be no visual control of other bodily movements.

Historical

Accurate measurements of the time-relations of eye-movements have been made. Noteworthy among these is the work of Huey (4) who used a light mechanical recording device, actuated by an attachment made directly to the eyeball. Dodge (5) used a method of photographically recording the movements of the corneal image of a light-source, and quite recently, Miles and Shen (6), using a similar method, investigated the movements of the eyes of Chinese subjects reading Chinese characters. The first two of these, particularly, contain many references to earlier work on the processes involved in the act of reading, and discussions of many other phases of the problem than the duration of the fixation-pause. As to this last, the figures given are of interest here.

Huey states the average fixational pause in the reading of one passage as 191σ with a mean variation of 49σ, and for another subject as 108σ with an m.v. of 33σ. In a graphic record presented by Dodge the pauses vary from 40σ to 240σ. Miles and Shen investigated the reading of both vertically and horizontally aligned Chinese characters, and found the average pause to be from 260σ to 430σ, the general averages for eleven subjects about 320σ, and 20σ to 30σ longer for horizontal than for vertical reading. The mean variation averaged about $\frac{1}{3}$ of the duration of the stop.

The Present Work

The procedure consisted essentially in instructing the subject to fixate a point, *A*, then to shift his fixation to the right to a point *B*, 20° away in the visual field; and to note, on the way, what he could see at the point *O*, midway between *A* and *B*. The object at *O* was designed to measure the time the eyes remained in (approximate) fixation at that point, and consisted of a circle of twelve perforations, in an opaque black screen, which flashed out in succession at a known and controllable rate. The number of bright spots seen could be estimated and reported and is thus a measure of the time of fixation upon the object.

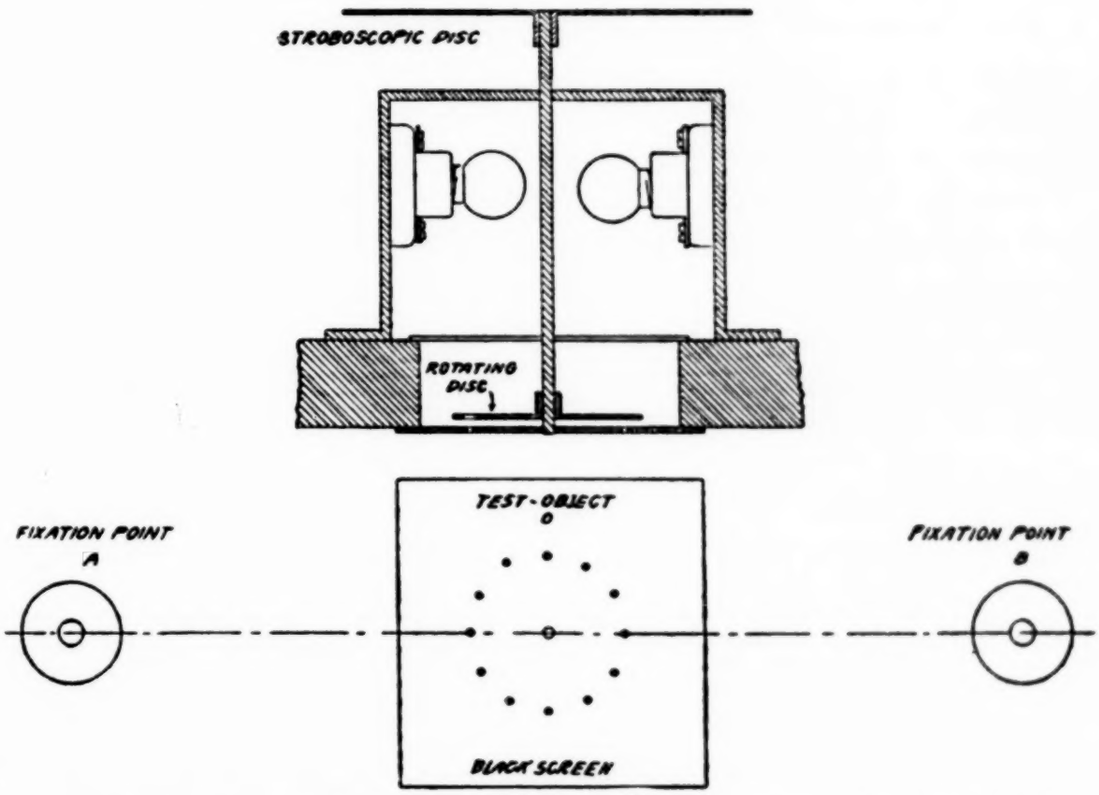

FIG. 1. The test-object. *A* = initial (red) fixation point. *B* = final (green) fixation point. *A* and *B* were 10° away from the test-object, which was placed at a distance of 2.85 m from the subject's eye. The circle on which the twelve openings were placed is 38 mm in diameter, the openings 2.5 mm.

Fig. 1 shows the test-object *O* as it appeared to the subject and a plan of its mechanism in section. The twelve perforations are arranged in a circle 38 mm in diameter subtending

a visual angle of 47.5 minutes at the subject's eye 2.85 meters away. The perforations are 2.5 mm in diameter. Directly behind the circle and concentric with it is a rotating disc with an opening near its edge wide enough to admit light through one perforation only at a time. Immediately behind the rotating disc is a milk-flashed glass diffusing screen, which is lit from behind by two miniature lamps. As the disc rotates the single opening in it passes behind each of the twelve in succession, permitting each to flash out in turn.

The subject was seated in a darkened room, a small spot of red light being visible at the first fixation-point, *A*. At a chosen time he pressed a key under his right hand, which extinguished the fixation-light *A* and lit up the test-object and the (green) fixation light *B*; and allowed his fixation to shift to *B*. The fact that he had been instructed also to see what he could at the test-object *O* resulted in a fixational pause at that point, as shown by the fact that he was able to report a definite number of bright spots at *O*, arranged in an arc of the circle. From this datum and the known speed of the disc it was possible to compute the time the eyes rested on the test-object. The test-object was too remote from *A* or *B* to permit clear vision with fixation at either point.

At first the task of reporting the number of spots seen presented some difficulty. One subject was never able to overcome this. For the greater part, however, the subjects became able to work rather rapidly, with only rare hesitation or failure to report.

It was found that monocular vision simplified the task. Accordingly, a rod was clamped to the head-rest used, extending in front of the left eye and cutting off from it all vision of test-object and both fixation-points. As the room was dark, this rod was invisible and did not disturb the subject in any way. The brightness of the fixation-spots and of the twelve openings in the test-object was made sufficient to avoid all uncertainty due to dimness but was kept low enough to avoid glare or other disturbance by reason of excess.

It is obvious that a speed of the disc that uncovered two

spots or less while the eyes paused would be impractical, and speeds high enough to uncover all twelve would yield meaningless results. The speed of the disc was therefore limited by the nature of the experiment. It was found practicable to operate the disc over a range of speed from 3.0 to 7.0 revolutions per second, and five speeds were used covering this range. The speed was checked by means of a stroboscopic disc mounted on the same shaft, and intermittently illuminated by a 10-watt lamp on the 60-cycle 115-volt power-circuit.

Nine subjects were used, each on five different days. The day's work consisted of five series of ten observations each, run with the five selected speeds of the disc respectively.

Results

In Table I are given the average results (in σ) for each subject at each speed of the disc. The averages for the first

Table I

Subj.	*N*	Revolutions of disc per sec					All speeds except 7.1		All speeds	
		3.0	3.5	5.0	6.0	7.1	Ave.	m.v.	Ave.	m.v.
1.....	5	146	149	150	116	124*a*	140		137	17
2.....	5	168	180	157	117	119	156		148	24
3.....	4	114	115	117	106	112	113		113	8
4.....	5	176	162	155*b*	121*a*	...*c*	154	17		
5.....	5	164	185	175*b*	144*a*	...*c*	167	18		
6.....	5	130	109	128	140	...*c*	127	15		
7.....	5	128	148	158	128	...*c*	140	18		
8.....	5	132	144	149	133	...*c*	140	13		
9.....	5	160	190	169	126	...*c*	161	24		
Ave....	1, 2, 3	143	148	141	113	118				
Ave....	all	146	154	151	126	...				

N, number of averages, each from a series of ten measurements, entering into the time-values in the body of the table. The times and mean variations (m.v.) are given in σ (thousandths of a second).

a, four values only involved in average.

b, speed of disc 4.3 revolutions per second.

c, complete circle of 12 spots usually seen; results not valid.

three subjects (yielding results for every speed used) are given in the next to the last line, and the averages for all subjects in

the last line. Similarly, the last two columns show the average results for each subject for all speeds. The mean variations of these averages which are given are conservative, since they are increased by the systematic differences, such as they are, between the values obtained at the different disc-speeds.

It will be noticed by a glance at the last two lines of the table that speeds of 3.0 to 5.0 revolutions per second of the disc yielded substantially identical results, averaging close to 150σ. At 6.0 and 7.1 revolutions the average duration of the pause is considerably shorter than this, by a difference little less than five times its probable error.

An attempt was made at the classification and plotting of some 1,000 of the results of single observations of the disc, involving the three speeds (3.0, 3.5 and 5.0 r.p.s.), which gave concordant results. The fact that the three speeds necessarily had to be plotted in different sets of abscissal (time) intervals made an accurate distribution curve impossible. However, by averaging the ordinates of the three curves (*A*, *B*, *C*, Fig. 2), it appeared that the entire distribution resembled the normal type, and was not far from symmetrical.

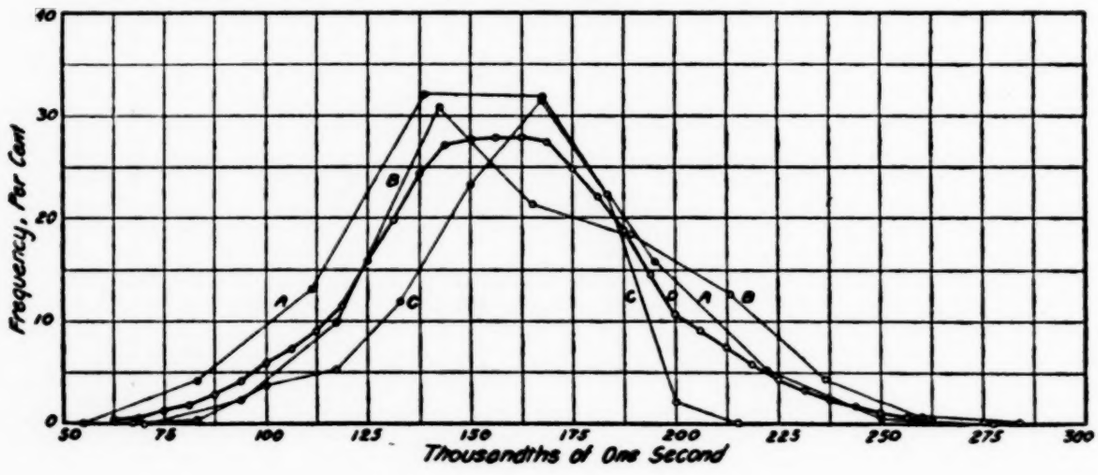

FIG. 2

Plotting the distributions for each disc-speed for Subject 3 also failed to show any distinct departure from symmetry, a fact which will be referred to later on.

Discussion

The method here used for measuring the fixational pause of the eyes has certain disadvantages and certain advantages as compared with the methods previously used by others. One disadvantage lies in its dependence upon the verbal report of an observation which Subject 3 (one of the authors), at least, found to be somewhat difficult and felt to be somewhat vague in the outcome. The method falls, perhaps, into the class sometimes called "subjective," as compared with one in which a graphic record is made for subsequent leisurely study and measurement. Further, the actual time-interval within which a given number, n, of spots may be uncovered, cannot be much more accurately defined than to say that it is more than the time corresponding to $n - 1$ intervals on the circle, and less than that corresponding to $n + 1$.

As to the advantages of this method, the one which appears to be most important is the fact that it makes possible the accumulation of a large number of results in a short time, without the risk and discomfort of making a mechanical attachment to the eyeball each time a subject comes to the experiment, and without the use of the expensive and troublesome photographic recording apparatus. In either case, the subsequent tedious work of measuring from the records and tabulating the measurements is avoided. Incidentally, the number of measurements actually involved in the averages of Table I is close to 1,900.

A further possible error to be mentioned is that of the selection of the results in the course of observation. One of the authors, Subject 3, found the work difficult and uncertain and he was unable to make report on many of his trials. In this case, we believe the results are selected, and the fact that this subject's average times are shorter than those of any others would seem to bear this out. On the other hand, the distribution of his results showed no definite asymmetry. Other subjects in the group, notably No. 1, were able to make, almost without exception, a definite report upon each trial.

A question of interest concerning the fixational pause of

the eyes is whether it may become shortened in the course of prolonged practice of a routine operation. The present results were examined with this in mind by the comparison of the averages within each group of four or five series for identical conditions. Thus in a series of five averages there are ten possible pairs of averages for comparison; within a group of four averages, six comparisons are possible. All told, the data yielded 358 such comparisons. In 154 of these, the later result was smaller than the earlier one, in 12 cases equal and in 192 cases the later result indicated a longer fixational pause than the earlier one. Thus these results, rather than showing a shortening of the pause with practice, actually showed a tendency for it to become longer as time went on. It must not be forgotten, however, that in no case was the opportunity for practice very great; no subject went through more than 25 series of ten separate observations each, and four of the subjects showed results contrary to the aggregate result just mentioned. The possibility that the time could be shortened by practice is still open. Indeed it would seem to be quite probable. In the study of the separate observations it appears that, while the average time was about 150σ, some few results were as low as 71σ and values of 83σ, while not frequent, occurred in considerable numbers. It is not unreasonable to suppose that the result of practice might in a measure prove to be that the occasional best performance of the unpracticed subject becomes in the course of time his usual performance.

Summary

1. A total of nearly 1,900 measurements, made with nine subjects, would indicate that the average fixational pause of the eyes, between two 10° sweeps, is close to 150σ.

2. The distribution of the separate measurements is not far from normal. Very few were as low as 71σ, and few as high as 250σ. The mean variation of the average from a series of ten observations runs as low as 8σ for one subject and as high as 24σ for two others. Selection of the results by the subject in the course of the work was not excluded by

the method used, and undoubtedly took place in the former case. In other cases it was probably negligible.

3. While the duration of the fixational pause tended on the whole to lengthen rather than to shorten as the experimental work proceeded, the possibility of materially shortening the average time by practice is by no means excluded.

References

1. Hess & Harrison. *Trans. Illum. Eng. Soc.*, 1923, **18**, 787–794.
2. Cobb, P. W. *J. Exper. Psychol.*, 1923, **6**, 138–160; 1925, **8**, 77–108; 1926, **9**, 95–109.
3. Ferree, C. E., & Rand, G. *Trans. Illum. Eng. Soc.*, 1922, **17**, 75–77.
4. Huey, E. *Amer. J. Psychol.*, 1900, **11**, 283–302.
5. Dodge, R. *Psychol. Rev. Monog. Suppl.*, 1907, no. 35, 95; *Zsch. f. Psychol.*, 1909, **52**, 321–422.
6. Miles & Shen. *J. Exper. Psychol.*, 1925, **8**, 344–362.

A STUDY OF THE PHENOMENON OF REMINISCENCE

BY OSBORNE WILLIAMS

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Historical Introduction

The purpose of this investigation is to study the phenomenon of reminiscence as a function of the age of the subjects and of the character of the material used. The nature of the study can best be stated in terms of its historical setting.

In 1885, Ebbinghaus (4), showed that the ability to recall nonsense material decreases with the passing of time and that the most rapid forgetting occurs immediately after learning. These results were later confirmed by Radossawljewitsch (9), Finkenbinder (5), Strong (10), Bean (2), and others, who found the relative amount forgotten, however, to be less than Ebbinghaus reported.

The original Ebbinghaus curve was obtained with one adult subject, with nonsense syllables, and with the relearning method of measuring the amount retained. The later experimenters employed various experimental conditions. They made use of poetry, abstract words, consonants and typewriting, in addition to nonsense materials. The methods of anticipation, recognition, selection and reconstruction, and written reproduction were also employed.

In spite of the variety of conditions the typical Ebbinghaus curve was invariably obtained. This curve was, therefore, generally assumed to represent a universal law of forgetting, although it was admitted that differences in the degree of learning, distribution of effort, length or amount of material, method of presentation, method of measuring the amount of retention, *etc.*, might affect the amount retained.

In 1913, however, Ballard (1) secured certain curves which differed from those of the Ebbinghaus type. The characteristic feature of them was the initial rise which occurred during the first two or three days after learning where the Ebbinghaus curve indicated a rapid decrease in retention. Ballard called this phenomenon of improvement in memory without further learning "reminiscence,"[1] as opposed to "obliviscence," or the process of decline in the capacity to remember.

[1] This use of the term is perhaps unfortunate inasmuch as reminiscence usually refers to any act of recall. We have decided to retain the term, however, because of its adoption by other writers.

Ballard experimented most on twelve-year-old children, and his percentages of retention for this age-level are shown in our Table *A*. Exhibit 1 was obtained from a 20 per cent. degree of learning of some verses from "The Wreck of the Hesperus;" Exhibit 2 represents a 40 per cent. degree of learning of parts of "The Ancient Mariner;" while Exhibit 3 is for 20 per cent. degree learning of nonsense verses. The amount remembered immediately after learning was taken as the basis on which the percentage later reproduced was calculated.

TABLE *A*

BALLARD'S RETENTION IN CHILDREN

Exhibits	Percentages Retained After Intervals of						
	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days
1.....	111.00	117.20	112.50	111.90	111.10	99.00	93.70
2.....	98.40	109.40	106.10	98.00	94.20	91.30	87.90
3.....	92.00	108.70	110.10	76.30	78.60	76.00	72.30

These peculiar findings of Ballard have since been verified by Huguenin (6), Brown (3), and Nicolai (8). Reminiscence must thus be regarded as a valid phenomenon.

This difference of results in the two sets of experiments is presumably due to some difference in the experimental conditions, which may be compared in four respects as follows.

1. The age of the subjects. While Ballard worked both with adults and children, he obtained the phenomenon of reminiscence only with children. Hence he assumed that reminiscence is a function of age. This conclusion is not justified, first because Ballard employed too few adult subjects and secondly because he varied the kind of material, the method of presentation, the degree of learning, and the method of measuring the amount of recall. Both Brown and Nicolai obtained reminiscence with adult subjects, and Radossawljewitsch secured the Ebbinghaus type of curve with children. Thus the results are not at all decisive.

2. The nature of the material. The experimental data furnish no evidence that reminiscence is a function of the kind of material employed. While Ballard utilized both connected and disconnected materials, and secured reminiscence only with such connected materials as ballad poetry and nonsense verse, yet Brown and Nicolai obtained the phenomenon with such disconnected materials as lists of abstract words, geographical names and familiar objects. Likewise, the Ebbinghaus type of curve has been secured with the use of poetry, as well as with nonsense syllables, abstract words and consonants.

3. The degree of learning. On *a priori* grounds it would seem that partial learning is an essential condition of the phenomenon, for reminiscence would hardly be possible with complete learning. This assumption is apparently justified by the data, for Ballard, Huguenin, Brown and Nicolai secured their results when the materials were only partially learned; while the previous investigators required their subjects to memorize the materials perfectly. But Ballard also obtained the Ebbinghaus type of curve when the materials were incompletely learned. Evidently some other coöperative factor is essential to the phenomenon.

4. The method of measuring the amount of retention. Reminiscence appears not to be a function of the method of measuring retention. Ballard, Huguenin, Brown and Nicolai used the method of written reproduction; but the Ebbinghaus declining curve has been obtained with this method as well as with the method of recognition, selection, anticipation and relearning.

Luh (7) recently performed a series of experiments designed to test the hypothesis that reminiscence is a function either of the degree of learning or of the method of measuring the amount retained. Employing nonsense syllables and adult subjects, he independently and systematically varied these two factors by using degrees of learning of 33 per cent., 67 per cent., 100 per cent. and 150 per cent., respectively, and by measuring the amount of retention by the methods of relearning, anticipation, recognition, reconstruction and written reproduction. All his curves conformed to the Ebbinghaus type, and he concluded that the difference between the Ebbinghaus tradition and the facts discovered by Ballard was not due to differences of either hypothetical factor.

The present study may be regarded as a supplement to that of Luh. It was designed to test the hypothesis that the phenomenon of reminiscence is a function of the age of the subjects and of the kind of material employed.

Method and Procedure

In our investigation two types of material were employed; (1) meaningful or connected material and (2) disconnected material. The first 40 lines of "The Spider and the Fly" by Mary Howitt were used in the first experiment, and in the second a list made up of 50 abstract monosyllabic words, 25 three-letter words and 25 four-letter words, all irregularly mixed. Care was taken in selecting these words to exclude all nouns and all words that could be readily associated, either directly or indirectly, with one another. We used words beginning with most of the letters of the alphabet, words with about the same degree of familiarity and only words in common usage. These materials were printed on cardboard to facilitate handling and to give each subject an individual copy.

We used 4,209 subjects, who were divided into four groups on the basis of age. In the first experiment 2,006 subjects were divided as follows: Age-group 1, 514 with an average age of 9.6 years; Age-group 2, 607 with an average age of 12.7 years; Age-group 3, 496 with an average age of 16.2 years, and Age-group 4, 389 adults. In the second experi-

ment were 2,203 subjects: 646 in Age-group 1 with an average age of 9.7 years; 621 in Age-group 2 with an average age of 12.8 years; 552 in Age-group 3 with an average age of 16.2 years, and 384 adults in Age-group 4.

The subjects in Age-groups 1, 2 and 3 were pupils in the Chicago Public Schools. The adult subjects of Age-group 4 were all college students. Age-group 1 consisted of third and fourth grade pupils, Group 2 of sixth and seventh grade pupils, while Group 3 contained second and third year high-school students.

Each of the four age-groups was further divided into five sections on the basis of the length of interval for which retention was tested. No subject was used in more than one section. We felt that about 100 cases in each section would be a sufficient number to make our conclusions statistically valid.

The subjects were tested during regular class periods, the experimenter conducting all tests in order to keep the method of presentation uniform. General information concerning the nature of the test was given while sheets of writing paper were being distributed. After the subjects had recorded their names, ages, *etc.*, on the sheets, a card with the printed material was placed face-down before each. Before the material was exposed the subjects were told (1) that on presentation the material would first be read aloud to them, (2) that they were then to learn or memorize as much of it as they could in five minutes, (3) that they must turn the cards face-down at the end of the learning-period and immediately write down on the sheets of paper as much as they could recall, (4) that they must try to reproduce the exact words as well as the thought, and (5) that they should try to put the material in the correct order although this was not essential. The experimenter collected the cards while the subjects were writing.

There was no definite time-limit for reproduction, although for each age-group the time remained fairly constant. Enough time was given for the subjects to recall as much of the material as they could. The younger subjects needed more time than the older. When the papers were collected the impression was given that the experiment was finished and the subjects did not once suspect a later recall.

Preliminary tests indicated that a short learning-period was desirable because (1) greater interest could be maintained, (2) the results would be less affected by fatigue, and (3) the tests could be completed during one class-period. We used a five-minute learning-period for both kinds of material and for all age-groups. In order to secure approximately a 50 per cent. degree of learning for each age-group we varied the amount of material. In the first experiment we provided 24 lines of poetry for Group 1, 32 lines for Group 2, and 40 lines each for Groups 3 and 4. In the second experiment 30 words were set for Group 1, 40 words for Group 2, and 50 words each for Groups 3 and 4.

Two written reproductions were required of all subjects; one immediately after learning and another after the interval for which we desired to test retention. We used intervals of 1, 2, 3, 5 and 7 days for both kinds of material and for all age-groups. One section of each age-group had its second recall after 1 day; another section after 2 days; another after 3; another after 5; and another after 7 days. In each school we arranged to have all the second reproductions on the same day in order to prevent our subjects from learning from each other that this second recall was required.

For the sake of uniformity all the scoring was done by the experimenter. The

list of abstract words was scored for the number correctly reproduced. The recall for poetry was scored in three ways, namely, (1) the number of lines reproduced perfectly, (2) the number reproduced as to sense, and (3) the number reproduced as to sense or part sense. These three methods are not opposed to each other, for the second method includes the first, and the third the first and second. We discarded the records of those subjects who had previously learned this poem or who admitted that they read it during the interval between the learning and the second reproduction.

Reminiscence as Related to Age and Material

This section presents the percentages of retention for the various time-intervals, for the four age-groups, and for the two types of material.

Table I

Poetry

Age-groups	Percentages Retained After Intervals of				
	1 Day	2 Days	3 Days	5 Days	7 Days
	Retention Scored as Correct Verbal Reproduction				
1.....	114.15	109.23	105.98	97.30	113.93
2.....	102.60	104.61	95.40	100.65	90.97
3.....	96.82	94.23	93.99	86.68	91.39
4.....	91.28	87.17	78.12	79.72	70.95
	Retention Scored as Reproduction of Sense				
1.....	106.65	103.01	104.01	98.83	105.25
2.....	107.03	101.64	99.29	97.47	94.02
3.....	96.28	94.65	92.56	91.61	91.59
4.....	96.44	96.46	89.76	91.02	80.11
	Retention Scored as Sense or Partial Sense				
1.....	105.22	102.68	106.76	97.06	108.35
2.....	106.99	101.04	96.60	96.49	94.06
3.....	96.15	94.26	90.89	87.62	90.21
4.....	96.50	94.10	90.91	88.64	82.49

We shall speak, hereafter, of the scores of the first reproduction as *amount learned*, and of the results of the second reproduction as *amount retained*.

The *percentages of retention* were computed by dividing the amounts retained by the amounts learned. In case fewer lines or words were retained than were learned the percentage

scores are less than 100; but if more were retained than were learned the percentage scores exceed 100. These percentage scores are given in Tables I and II, and graphically represented in Figures 1 and 2.

TABLE II

WORDS

Age-groups	Percentages Retained After Intervals of				
	1 Day	2 Days	3 Days	5 Days	7 Days
	Retention				
1.....	69.60	72.78	68.27	59.72	55.09
2.....	76.45	68.27	68.99	65.40	49.85
3.....	77.02	72.68	67.17	58.39	58.37
4.....	77.89	73.81	67.58	65.01	57.18

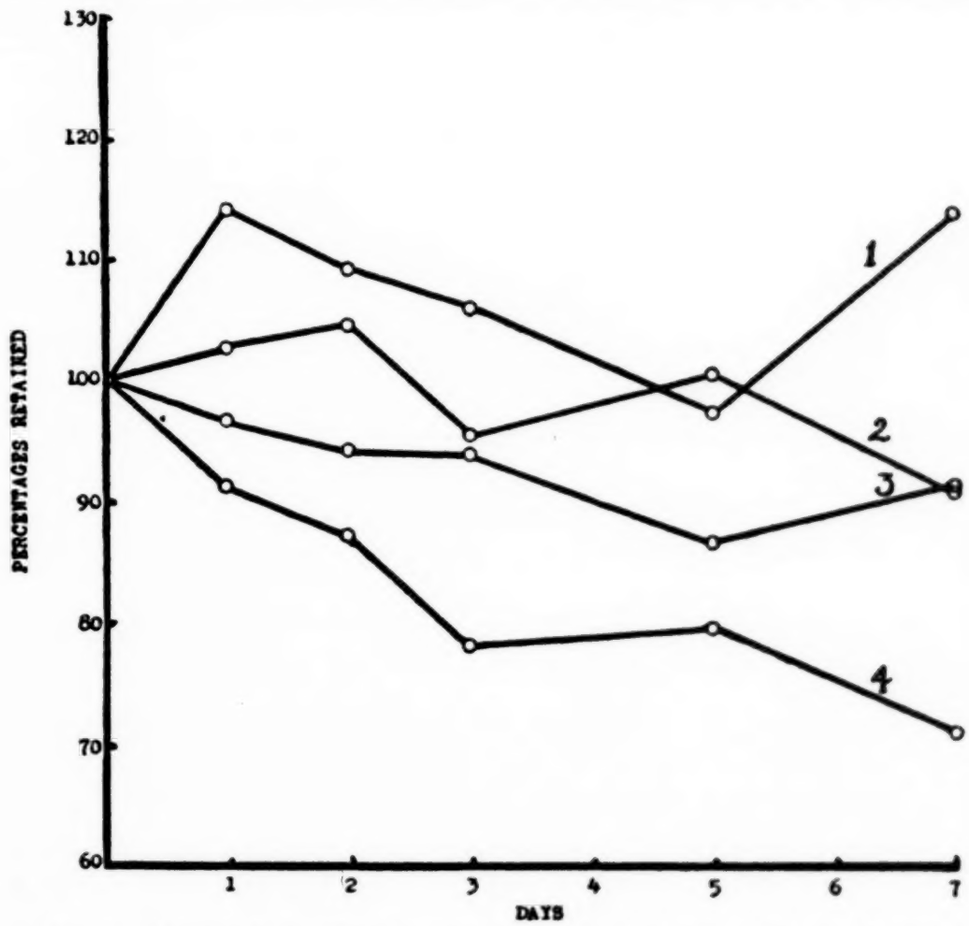

FIG. 1. Curves of retention for poetry, correct verbal reproduction. Curves 1, 2, 3, and 4 are for Age-groups 1, 2, 3, and 4 (ages 9+, 12+, 16+, and 21+), respectively.

No reminiscence was exhibited by any of the age-groups when lists of *words* were used as the material. The four retention curves are practically identical. In other words, the degree of retention is not a function of the age of the subjects. The four curves are similar to those obtained by Finkenbinder and Radossawljewitsch.

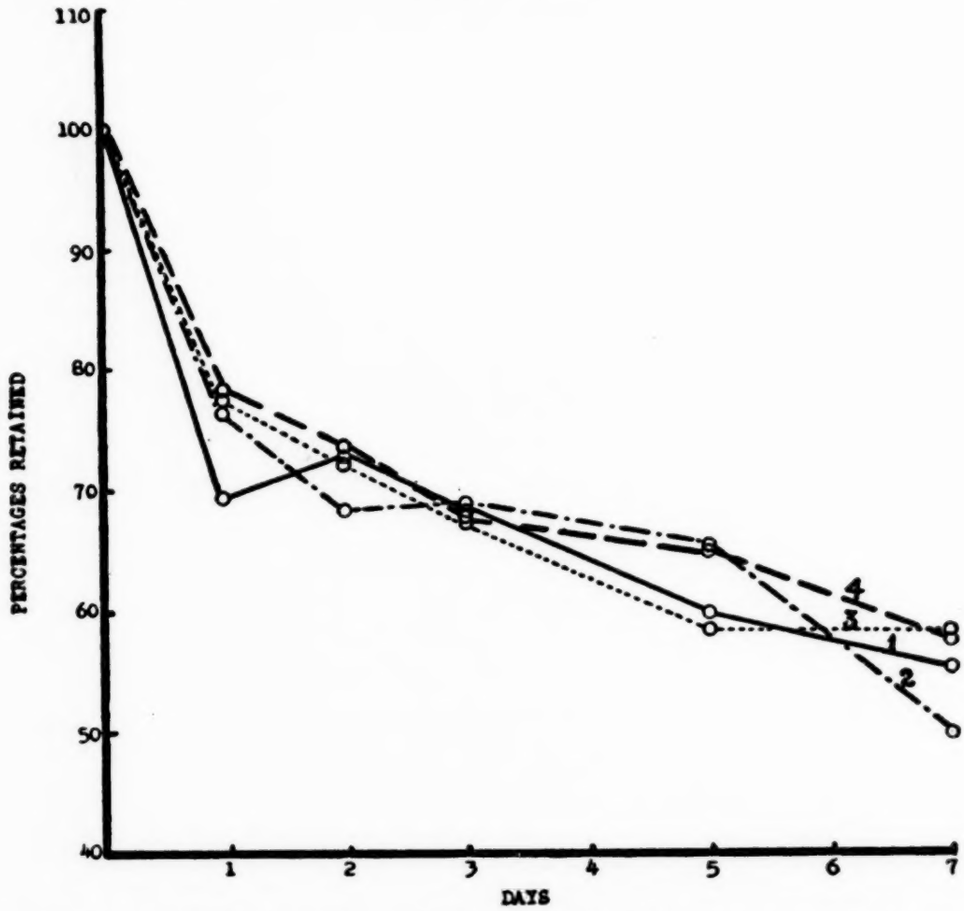

FIG. 2. Curves of retention for abstract words. Curves 1, 2, 3, and 4 are for Age-groups 1, 2, 3, and 4 (ages 9+, 12+, 16+, and 21+), respectively.

When *poetry* was employed as the material, the degree of retention was inversely related to the age of the subjects. Reminiscence was exhibited by the two younger age-groups, and the greatest amount and duration of reminiscence were displayed by the youngest age-group. The curves for the two older age-groups conform to the Ebbinghaus type but exhibit a less rapid rate of forgetting than that usually obtained. For all age-groups the rate of forgetting is less

for poetry than that secured with lists of words. The degree of retention for all age-groups is, then, a function of the material.

The percentages retained vary to some extent with the method of scoring. The adults made better records when scored on the basis of the amounts reproduced as to sense. The two middle age-groups did equally well when scored as to sense and for correct verbal reproduction. The youngest children, however, made better records when scored in terms of correct verbal reproduction. With the youngest children, the degree of reminiscence is thus to a slight extent a function of the method of scoring.

TABLE III

AVERAGE NUMBER OF *Lines* PERFECTLY REPRODUCED WITH PERCENTAGES OF RETENTION

	Thirds	Individuals		Average Amounts		Per-cent-ages
		No.	Age	Learned	Retained	Retained
Group I.....	Lowest.....	170	9.9	1.88	2.69	142.81
	Middle.....	170	9.7	4.49	4.73	105.23
	Highest.....	174	9.4	8.38	8.65	103.22
Group II.....	Lowest.....	202	12.9	4.25	4.56	107.46
	Middle.....	201	12.8	8.16	8.08	99.02
	Highest.....	204	12.4	13.58	13.13	96.71
Group III.....	Lowest.....	167	16.4	6.90	6.59	95.49
	Middle.....	163	16.2	11.75	10.88	92.53
	Highest.....	167	15.9	18.11	16.72	92.33
Group IV.....	Lowest.....	130	Adult	8.88	7.53	84.76
	Middle.....	128	"	14.52	11.47	78.97
	Highest.....	131	"	22.63	18.93	83.64

In order to study the relationship between the amount of reminiscence and the degree of learning within each age-group, we divided each of the four age-groups into three sub-groups on the basis of the amount learned. These data are presented in Tables III and IV, where are given for each of these sub-groups the number of individuals, their average

age, the average amounts learned and retained, and the percentage retained.

TABLE IV

AVERAGE NUMBER OF *Words* REPRODUCED WITH PERCENTAGES OF RETENTION

	Thirds	Individuals		Average Amounts		Per-cent-ages
		No.	Age	Learned	Retained	Retained
Group I.....	Lowest.....	215	9.7	8.86	6.44	72.74
	Middle.....	213	9.7	12.48	7.96	63.67
	Highest.....	218	9.7	16.52	10.40	62.98
Group II.....	Lowest.....	206	12.9	14.01	9.70	69.26
	Middle.....	206	12.6	18.38	12.28	66.80
	Highest.....	209	12.7	23.21	14.96	64.47
Group III.....	Lowest.....	184	16.5	18.24	12.09	66.30
	Middle.....	183	16.2	24.02	15.87	66.08
	Highest.....	185	15.9	30.07	20.28	67.45
Group IV.....	Lowest.....	128	Adult	20.40	13.77	67.48
	Middle.....	127	"	26.89	18.17	67.55
	Highest.....	129	"	33.48	23.83	71.17

Within each age-group, the percentage of retention varied inversely with the amount learned, and directly with the age of the subjects for correct verbal reproduction of the poetic material; while, with the list of words, the percentage of retention varied inversely with the amount learned for the two younger age-groups; but no correlation is present for the two older age-groups. In general, there is no correlation between the percentage of retention and the age of the subjects for the latter type of material. Within the two younger age-groups the *degree of reminiscence* is thus a *function of the amount learned.*

The data thus justify the following conclusions:

(1) The phenomenon of reminiscence is a function of the age of the subjects and also of the material memorized. The degree of reminiscence is slightly dependent upon the method of scoring. Within a given age-group the greatest amount of reminiscence is exhibited by the poorest learners.

(2) The degree of retention for all age-groups is a function of the material.

(3) For certain materials the degree of retention is inversely related to the age of the subjects; but for other materials it is independent of age.

(4) Within each age-group the degree of retention is inversely related to the amount learned and directly related, for certain materials, to the age of the subjects; while for other materials the degree of retention bears no definite relationship either to age or to the degree of learning.

(5) The relation of age to the retention of poetry varied with the method of scoring.

Sex Differences in Learning and Retention

Here are presented the average learning- and retention-scores for both experiments. The data for the four age-groups and for the three methods of scoring the *poetic* material are given in Tables V to VIII; the scores for the *word-lists* in Table IX.

Table V

Age-Group I

Intervals	Boys	Girls	Ages	Amounts Learned			Amounts Retained		
				Boys	Girls	Totals	Boys	Girls	Totals
Average Number of Lines Perfectly Reproduced									
1 day.....	49	57	10.27	5.49	6.48	6.22	6.02	8.04	7.10
2 days.....	52	50	9.39	4.87	5.32	5.09	5.44	5.68	5.56
3 "	55	45	8.90	3.38	4.04	3.68	3.62	4.24	3.90
5 "	46	60	9.56	3.37	5.28	4.45	3.17	5.22	4.33
7 "	46	54	9.84	4.50	5.87	5.24	4.96	6.83	5.97
	248	266	9.60	4.31	5.53	4.95	4.64	6.07	5.38
Average Number of Lines Reproduced as to Sense									
1 day.....	49	57	10.27	14.10	15.02	14.59	14.63	16.35	15.56
2 days.....	52	50	9.39	10.87	12.40	11.62	11.29	12.68	11.97
3 "	55	45	8.90	7.78	9.33	8.48	8.11	9.69	8.82
5 "	46	60	9.56	9.20	9.57	9.41	8.54	9.88	9.30
7 "	46	54	9.84	10.28	11.69	11.04	10.70	12.41	11.62
	248	266	9.60	10.40	11.66	11.05	10.63	12.27	11.28

Average Number of Lines Reproduced as to Sense or Partial Sense

1 day.....	49	57	10.27	15.39	16.00	15.72	15.92	17.07	16.54
2 days.....	52	50	9.39	11.69	13.00	12.33	11.92	13.42	12.66
3 "	55	45	8.90	8.15	9.75	8.87	8.67	10.44	9.47
5 "	46	60	9.56	10.30	10.15	10.22	9.35	10.37	9.92
7 "	46	54	9.84	10.80	12.30	11.61	11.63	13.39	12.58
	248	266	9.60	11.21	12.31	11.78	11.46	13.00	12.26

TABLE VI

AGE-GROUP 2

Intervals	Boys	Girls	Ages	Amounts Learned			Amounts Retained		
				Boys	Girls	Totals	Boys	Girls	Totals

Average Number of Lines Perfectly Reproduced

1 day.....	76	77	12.45	6.79	8.57	7.69	6.83	8.95	7.89
2 days.....	60	61	12.40	7.33	10.41	8.89	7.65	10.93	9.30
3 "	55	58	13.15	8.15	12.60	10.43	7.42	12.36	9.95
5 "	55	59	12.55	6.85	8.44	7.67	6.64	8.73	7.72
7 "	57	49	12.91	7.40	11.04	9.08	6.63	10.37	8.26
	303	304	12.70	7.27	10.08	8.68	7.03	10.18	8.61

Average Number of Lines Reproduced as to Sense

1 day.....	76	77	12.45	13.29	14.86	14.08	14.22	15.90	15.07
2 days.....	60	61	12.40	15.87	19.41	17.65	16.27	19.59	17.94
3 "	55	58	13.15	16.27	20.05	18.21	15.91	20.14	18.08
5 "	55	59	12.55	14.36	16.42	15.43	13.78	16.22	15.04
7 "	57	49	12.91	15.33	18.73	16.90	14.18	17.88	15.89
	303	304	12.70	14.92	17.69	16.31	14.84	17.83	16.34

Average Number of Lines Reproduced as to Sense or Partial Sense

1 day.....	76	77	12.45	13.96	15.23	14.60	14.78	16.45	15.62
2 days.....	60	61	12.40	18.93	21.33	20.14	19.03	21.64	20.35
3 "	55	58	13.15	19.04	22.01	20.59	17.98	21.71	19.89
5 "	55	59	12.55	15.91	17.67	16.82	15.18	17.20	16.23
7 "	57	49	12.91	17.89	20.67	19.18	16.68	19.61	18.04
	303	304	12.70	16.96	19.11	18.04	16.63	19.15	17.89

TABLE VII

AGE-GROUP 3

Intervals	Boys	Girls	Ages	Amounts Learned			Amounts Retained		
				Boys	Girls	Totals	Boys	Girls	Totals
Average Number of Lines Perfectly Reproduced									
1 day.....	48	56	16.23	11.40	14.79	13.22	11.40	14.00	12.80
2 days.....	42	60	15.73	11.31	13.28	12.47	10.21	12.83	11.75
3 "	38	74	16.24	10.97	13.27	12.49	11.05	12.09	11.74
5 "	54	41	16.47	10.59	11.98	11.19	8.81	10.88	9.70
7 "	35	48	16.13	10.89	12.56	11.85	10.31	11.21	10.83
	217	279	16.20	11.02	13.27	12.28	10.29	12.30	11.42
Average Number of Lines Reproduced as to Sense									
1 day.....	48	56	16.23	19.67	23.59	21.78	18.88	22.77	20.97
2 days.....	42	60	15.73	19.86	23.93	22.25	18.45	22.88	21.06
3 "	38	74	16.24	20.32	23.34	22.31	18.87	21.57	20.65
5 "	54	41	16.47	18.20	20.24	19.08	15.85	19.63	17.48
7 "	35	48	16.13	19.51	21.96	20.93	17.03	20.73	19.17
	217	279	16.20	19.43	22.82	21.34	17.74	21.66	19.95
Average Number of Lines Reproduced as to Sense or Partial Sense									
1 day.....	48	56	16.23	21.33	25.18	23.40	20.21	24.46	22.50
2 days.....	42	60	15.73	21.76	25.37	23.88	20.17	24.15	22.51
3 "	38	74	16.24	23.18	25.26	24.55	20.89	23.04	22.31
5 "	54	41	16.47	21.41	23.27	22.21	17.98	21.41	19.46
7 "	35	48	16.13	21.11	23.29	22.37	17.89	21.85	20.18
	217	279	16.20	21.72	24.63	23.36	19.39	23.12	21.49

These tables present for each of the sections of the four age-groups, and for each age-group as a whole, the following results. The interval for which retention was tested; the total number of individuals (including the number of each sex with average age), and the average amounts learned and retained by the two sexes taken both separately and together. The learning- and retention-scores for the two sexes, for the four age-groups, and for the correct verbal reproductions for poetry are graphically represented in Figure 3. Similar scores for the word-lists are graphically represented in Fig. 4.

TABLE VIII

AGE-GROUP 4

Intervals	Boys	Girls	Ages	Amounts Learned			Amounts Retained		
				Boys	Girls	Totals	Boys	Girls	Totals
Average Number of Lines Perfectly Reproduced									
1 day.....	35	42	Adult	13.97	17.17	15.71	12.37	15.98	14.34
2 days.....	26	70	"	13.65	19.63	18.01	11.12	17.40	15.70
3 "	32	37	"	12.69	14.59	13.71	9.28	11.95	10.71
5 "	44	32	"	13.16	14.34	13.66	10.25	11.78	10.89
7 "	55	16	"	14.02	17.81	14.87	9.80	13.13	10.55
	192	197	Adult	13.54	17.15	15.37	10.46	14.81	12.67
Average Number of Lines Reproduced as to Sense									
1 day.....	35	42	Adult	23.54	26.76	25.30	22.69	25.83	24.40
2 days.....	26	70	"	23.38	28.54	27.14	21.92	27.77	26.18
3 "	32	37	"	20.00	23.27	21.77	17.13	21.62	19.54
5 "	44	32	"	21.84	23.97	22.73	19.52	22.31	20.69
7 "	55	16	"	21.47	26.19	22.53	16.89	22.06	18.05
	192	197	Adult	21.95	26.24	24.12	19.27	24.85	22.10
Average Number of Lines Reproduced as to Sense or Partial Sense									
1 day.....	35	42	Adult	26.66	29.10	27.99	25.71	28.10	27.01
2 days.....	26	70	"	26.73	30.99	29.83	24.77	29.30	28.07
3 "	32	37	"	20.75	23.68	22.32	18.19	22.11	20.29
5 "	44	32	"	23.59	27.34	25.17	20.75	24.47	22.31
7 "	55	16	"	23.33	27.56	24.28	18.82	24.19	20.03
	192	197	Adult	24.03	28.34	26.21	21.22	26.49	23.89

In each age-group the girls invariably excelled the boys in learning ability for both types of material as well as for the three methods of scoring. Comparing the various sections, the girls excelled the boys in seventy-nine of the eighty cases.

This superiority of the girls increased slightly with age, but the increase was not proportional either to age or to the amounts learned. For the two younger age-groups, the superiority of the girls did not vary with the method of scoring. For the two older age-groups, however, the girls

TABLE IX

AVERAGE NUMBER OF WORDS REPRODUCED

Intervals	Boys	Girls	Ages	Amounts Learned			Amounts Retained		
				Boys	Girls	Totals	Boys	Girls	Totals
Age-Group I									
1 day.....	73	69	9.61	12.30	12.54	12.42	8.49	8.80	8.64
2 days.....	67	67	10.18	12.79	13.81	13.30	8.93	10.43	9.68
3 "	58	63	9.52	11.83	13.03	12.45	8.14	8.83	8.50
5 "	71	63	9.99	12.31	13.40	12.82	7.15	8.22	7.66
7 "	54	61	9.31	11.59	12.59	12.12	6.54	6.80	6.68
	323	323	9.70	12.20	13.07	12.64	7.90	8.65	8.28
Age-Group II									
1 day.....	73	71	12.62	17.27	19.01	18.13	12.89	14.86	13.86
2 days.....	62	62	12.26	18.77	21.03	19.90	12.47	14.71	13.59
3 "	47	74	12.87	17.64	18.61	18.23	11.74	13.11	12.58
5 "	58	62	13.00	17.26	19.15	18.23	11.29	12.48	11.93
7 "	57	55	12.89	16.81	19.85	18.30	7.86	10.44	9.13
	297	324	12.80	17.55	19.48	18.56	11.34	13.23	12.33
Age-Group III									
1 day.....	51	57	15.97	23.45	26.58	25.10	16.98	21.44	19.33
2 days.....	61	47	16.67	24.02	25.36	24.60	17.13	18.85	17.88
3 "	66	49	16.07	21.18	23.31	22.09	13.65	16.43	14.83
5 "	52	61	15.87	23.02	27.49	25.43	12.46	16.89	14.85
7 "	46	62	16.36	22.80	23.92	23.44	13.87	13.55	13.69
	276	276	16.20	22.48	25.39	24.12	14.85	17.33	16.09
Age-Group IV									
1 day.....	42	41	Adult	25.93	29.71	27.80	19.74	23.61	21.65
2 days.....	31	56	"	25.90	28.82	27.78	18.00	21.89	20.51
3 "	38	34	"	26.42	29.79	28.01	17.74	20.29	18.93
5 "	26	46	"	22.46	25.70	24.53	13.69	17.22	15.94
7 "	29	41	"	21.14	29.88	26.26	13.62	16.00	15.01
	166	218	Adult	24.66	28.68	26.94	16.94	19.87	18.60

exhibited a slightly greater degree of superiority when the material was scored for sense reproduction; but this difference was not proportional to age nor to the amounts learned. Measured in both absolute and relative terms, there is no good evidence that this sex-difference in ability to learn varied with the kind of material employed. It is notably consistent for all ages, for both types of material and for the three ways of scoring the reproductions.

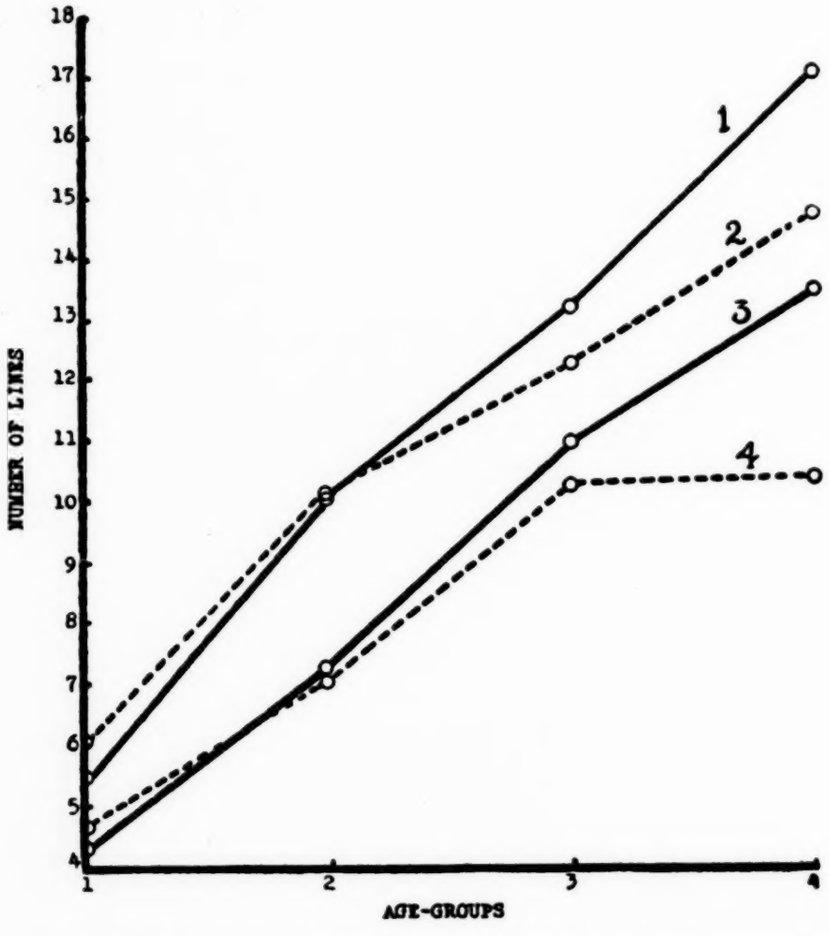

FIG. 3. Graphical representation of correct verbal reproductions for poetry: 1, learning for girls; 2, retention for girls; 3, learning for boys; 4, retention for boys.

With respect to the amounts of material retained, the girls again excelled the boys within each age-group, for both kinds of material and for the three methods of scoring, the girls surpassing the boys in seventy-nine of the eighty cases.

Once more, this superiority of the girls increased somewhat with age. The method of scoring did not materially

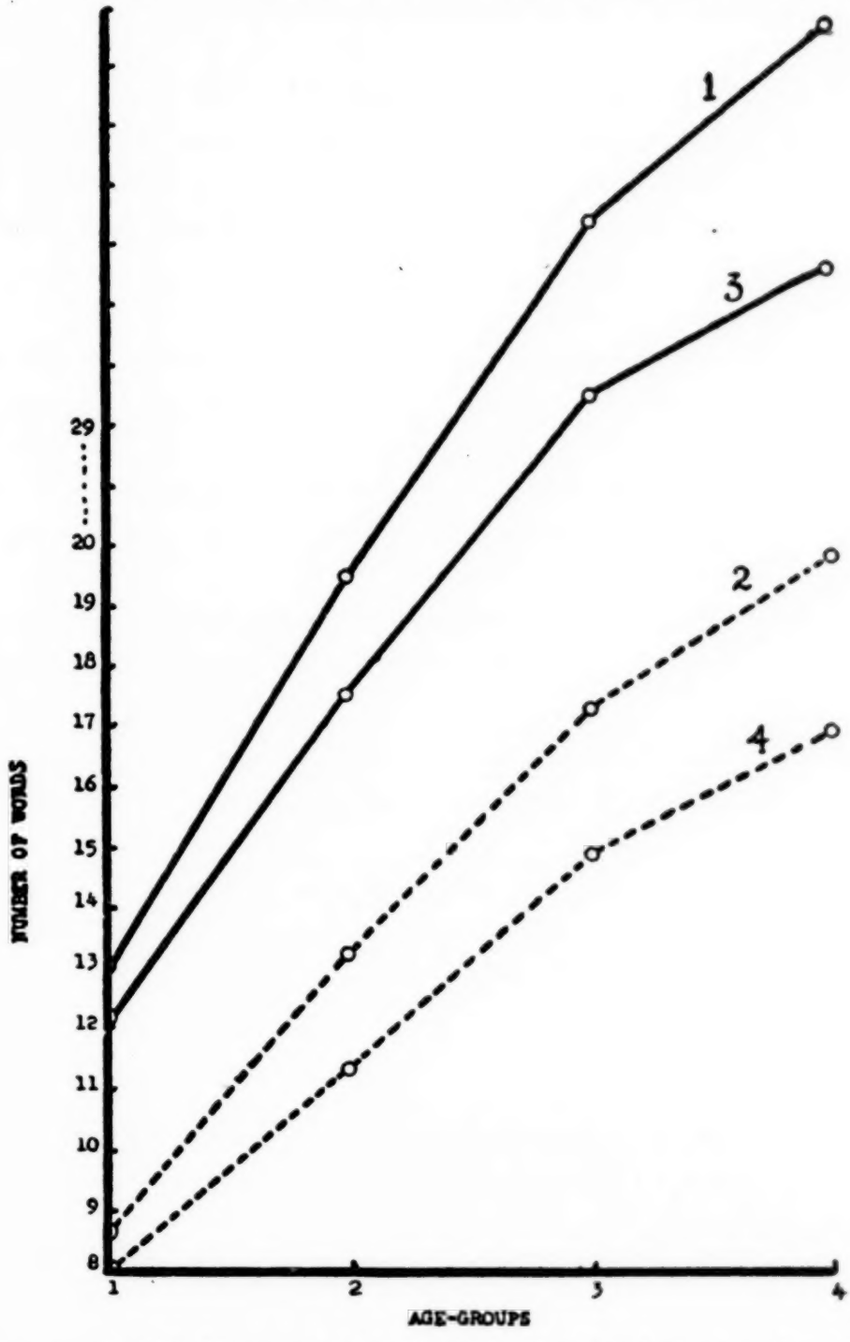

FIG. 4. Graphical representation of reproductions for abstract words: 1, learning for girls; 2, retention for girls; 3, learning for boys; 4, retention for boys.

affect it for the two younger age-groups. For the two older age-groups, however, the girls excelled the boys to a slightly greater extent when the reproductions were scored for sense or

partial sense than when scored for correct verbal reproduction. Sex-differences in the amounts retained varied with the type of material. These differences, in favor of the girls, are greater for poetry than for the verbal lists.

The superiority of the girls with respect to the amounts retained might naturally be inferred from their superior learning ability. But the girls also retained a slightly larger percentage of that which they learned. Their superiority in retention was greater than in learning.

The percentages of retention for the two sexes of the four age-groups, for the two types of material, and for the three ways of scoring the reproductions for poetry are given in Table X. These percentage scores were derived by dividing the amounts retained by the amounts learned.

Table X

Percentages of Retention

Age-groups	Words		Poetry					
			Reproductions; Correct Verbal		Reproductions; as to Sense		Reproductions; as to Sense or Partial Sense	
	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
1.....	64.75	66.18	107.66	109.76	102.21	105.23	102.23	105.61
2.....	64.62	67.92	96.70	100.99	99.46	100.80	98.05	100.21
3.....	66.06	68.22	93.38	92.69	91.30	94.92	89.27	93.87
4.....	68.70	69.28	77.25	86.36	87.79	94.70	88.31	93.48

It is evident from these data that the girls exhibited greater retentive ability for each age-group, for both types of material, and for the three methods of scoring. They were superior in fifteen of the sixteen comparisons.

This superiority of the girls varied slightly with age and with the material. The greatest amount of superiority was displayed by the girls of the two older age-groups in their ability to retain poetry.

The sexes also differed in respect to reminiscence. In the youngest age-group, the girls exhibited a greater degree of

reminiscence, while in Age-group 2 reminiscence occurred only for the girls. The degree of reminiscence as well as its frequency of occurrence are thus to some extent a function of sex.

Summary and Conclusions

Partial learning is obviously one of the essential conditions of the phenomenon of reminiscence. The phenomenon was exhibited only by the two younger age-groups, and for the connected type of material. The greater degree of reminiscence and reminiscence for the longer intervals of time were manifested by the youngest age-group. Within these age-groups, the subjects who learned the least amount of material exhibited the greatest degree of reminiscence. Moreover, these age-groups showed the greatest amount of reminiscence when the material was scored for correct verbal reproduction. Reminiscence occurred more frequently with girls than with boys, and girls displayed a greater degree of reminiscence when it was present in both sexes.

So far we have merely stated the conditions under which reminiscence occurred without making any attempt to explain it. Any suggested explanation must take into account the fact that reminiscence may be a special case of the more general law that the percentage of retention varies with the material, the method of scoring, the amount learned, and the age and the sex of the subjects.

It may be suggested that the inverse correlation between the percentage of retention and the age of the subjects is due to the fact that the younger children learned a smaller proportion of the material presented, and, hence, the opportunity for reminiscence was greater. Reminiscence, therefore, may be a function of the degree of learning rather than of some other factor connected with mental growth. The percentages of retention, however, will necessarily exhibit a spurious inverse correlation with the learning scores inasmuch as these percentage values were derived by dividing the amounts retained by the learning scores.

The degree of retention may be a function of the extent to which the material was spontaneously revived during the

interval for which retention was tested. It is possible that reminiscence for the poetic material was manifested by the younger children because its subject-matter and rhythmical construction naturally appealed to children of this age. Possibly these same children would not have exhibited reminiscence for other types of connected material.

The greater degree of reminiscence manifested by the girls may, likewise, be due in part to their greater interest in this type of material, or to the absence of various distracting conditions that would naturally inhibit the spontaneous arousal of the material during the interval. The fact that the children exhibited the greater degree of reminiscence when the material was scored for correct verbal reproduction may be due in part to their greater training in verbatim methods of memorizing.

The division of the age-groups on the basis of the amount learned does not necessarily represent differences in learning ability. Some of the subjects in the lowest third of the age-groups made poor scores on the first reproduction because of unfavorable chance conditions that obtained at the time of learning. Likewise, chance conditions necessarily favored certain of the individuals of the highest third of these age-groups. According to chance, many of the subjects in the lowest third would make relatively better scores in the second reproduction, while the individuals of the highest third would be likely to make relatively poorer records in the second test. This factor of chance may thus account in part for the greater reminiscence exhibited by the poorer learners.

These explanatory concepts are offered merely as suggestions. In all probability the factors accounting for reminiscence are many and various. Our experimental procedure was designed, primarily, to ascertain some of the factual conditions of the phenomenon rather than to explain it.

References

1. Ballard, P. B. Obliviscence and reminiscence, *Brit. J. Psychol., Monog. Suppl.*, 1913, I.
2. Bean, C. H. The curve of forgetting, *Arch. of Psychol.*, 1912, **20**, No. 3.
3. Brown, W. To what extent is memory measured by a single recall, *J. Exper. Psychol.*, 1923, **6**, 377–82.

4. EBBINGHAUS, H. *Über das Gedächtniss*, Leipzig, 1885. (Eng. Tr. by Ruger and Bussenius, Teachers' College, Columbia University, 1913.)
5. FINKENBINDER, E. O. The curve of forgetting, *Amer. J. Psychol.*, 1913, **24**, 8–32.
6. HUGUENIN, C. Reviviscence paradoxale, *Arch. d. Psychol.*, 1914, **14**, 379–83.
7. LUH, C. W. The conditions of retention, *Psychol. Rev., Monog. Suppl.*, **31**, No. 3.
8. NICOLAI, F. Experimentelle Untersuchungen über das Haften von Gesichtseindrücken und dessen zeitlichen Verlauf, *Arch. f. d. ges. Psychol.*, 1921, **42**, 132–49.
9. RADOSSAWLJEWITSCH, P. R. *Das Behalten und Vergessen bei Kindern und Erwachsenen*, Leipzig, 1907.
10. STRONG, E. K. The effect of time-interval upon recognition memory, *Psychol. Rev.*, 1913, **20**, 339–72.

RACIAL DIFFERENCES IN THE INTELLIGENCE OF SCHOOL CHILDREN

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The widespread interest in tests of intelligence which has sprung up in this country during recent years has led to a number of studies of the comparative performance of various immigrant groups and their immediate descendants. No attempt will here be made to review these studies in detail; but the general nature of the results of them with respect to children of school age has been summarized in the accompanying list. For the greater part these studies are based upon American-born children; although it is probable that in some instances foreign-born children have also been included.

The figures presented are representative of the findings of a number of other investigators whose results have not been included. They also correspond rather closely to the scores made by the different nativity groups in the Army Intelligence Tests as reported by Brigham (3). It should be emphasized that figures such as these have reference only to the representatives of the various nationality groups who have immigrated to this country, and that selective immigration may be in large measure responsible for the results which have been obtained. That these immigrant groups and their immediate descendants markedly differ in their performances in the ordinary type of intelligence-test is a fact which appears to have been reasonably well established.

Two theories have been offered to account for these differences. The first ascribes the inferior showing made by the South Europeans and the negroes to such post-natal factors as inferior environment, poor physical condition and linguistic handicaps. The second point of view, while it

Intellectual Ability of American School Children by Racial Stock

Racial Stock	Reported by	Cases	Test used	Result
White American	Pintner and Keller (28)	249	Stanford-Binet	Mean IQ 95
White American	Sheldon (34)	100	"	104
White American	Dickson (9)	49	"	106
American Negro (Ohio)	Pintner and Keller (28)	71	"	88
American Negro (Tennessee)	Peterson (27)	All children 8–10 yrs. in several schools	Pressey	75
American Negro (Arkansas)	Jordan (18)	247	N.I.T.	14-yr. negroes equal to 10-yr. whites
American Negro (Northern)	Thorndike (40)	349	I.E.R.	Ca. 4 per cent. negroes passed median score for whites of same grade
American Negro	Sunne (36)	Over 1,000	N.I.T. and Myers	Negro ave. 1–1½ yrs. below whites (mental age)
English	Pintner and Keller (28)	24	Stanford-Binet	Mean IQ 97
English	Brown (4)	90	"	101.8
Italian	Pintner and Keller (28)	313	"	84
Italian	Dickson (9)	25	"	84
Italian	Young (45)	Several hundred	Army Alpha and Beta	about 83
Italian	Brown (4)	51	Stanford-Binet	77.5
German	Pintner and Keller (28)	37	"	91
German	Brown (4)	67	"	102.3
Jewish	Pintner and Keller (28)	79	"	95
Jewish	Murdock (24)	Several hundred	Pressey	Jews approx. equal to white Americans
Chinese (San Francisco)	Young (44)	109	Stanford-Binet	Median IQ 97
Chinese (Hawaii)	Symonds (38)	513	Pintner non-language	Mean IQ 99
Spanish-Mexican	Sheldon (34)	100	Stanford-Binet and Cole-Vincent	89
Spanish-Mexican	Dickson (9)	37	Stanford-Binet	78

Racial Stock	Reported by	Cases	Test used	Result
Portuguese	Young (45) (Quoting unpub'ed data by Roll)	119	Stanford-Binet	86
Portuguese	Dickson (9)	23	"	84
Norwegian	Brown (4)	34	"	103.8
Swedish	Brown (4)	187	"	101.9
Austrian	Brown (4)	28	"	99.5
French	Brown (4)	199	"	95.4
Finnish	Brown (4)	226	"	90
Slavish	Pintner and Keller (28)	130	"	85
Hungarian	Pintner and Keller (28)	99	"	89
Indian (Michigan)	Rowe (32)	268	Goddard-Binet	Only 5.8 per cent. of Indians tested at age or above
Indian	Hunter (17)	711	Otis	Whites excel Indians by 1.6 P.E. of latter. Correl'n bet. degree of white blood and score = .51 ± .017

recognizes that the factors named may to some degree affect the test-results, nevertheless holds that it is impossible to account for all the facts which have been observed upon any other hypothesis than that of innate differences among the groups under consideration.

It is unquestionably true that the home surroundings of certain racial groups, notably the Italians and negroes, are, as a rule, far less favorable than those of average American children. Not only is this true of the foreign-born Italians, but their American-born descendants frequently continue to live in the same neighborhoods and with little or no improvement in social or hygienic conditions. In this respect a notable difference may be observed between the Italian and the Jew. Both find a home in the slum on first coming to this country; but while the Italian remains there, the Jew soon moves to a better neighborhood.

"Social pressure" or "race prejudice" is often urged as a reason for the segregation of certain racial groups within the poorer neighborhoods. In this connection it should be remembered that while racial prejudice may bring about segregation, the character of the neighborhoods thus set off is primarily dependent upon the people living within them. It is doubtful whether even the Southern negro has more to contend with in the way of this prejudice than the Chinese or Japanese in California; yet the contrast between the typical Oriental neighborhood and the Italian or negro district is marked. Poverty there may be; but the squalor which is characteristic of the Italian and the negro sections is lacking.

It seems probable, upon the whole, that inferior environment is an effect at least as much as it is a cause of inferior ability, as the latter is indicated by intelligence tests. The person of low intelligence tends to gravitate to those neighborhoods where the economic requirement is minimal; and, once there, he reacts toward his surroundings along the line of least resistance. His children inherit his mental characteristics.

The question of a possible handicap in language as a cause of low scores on intelligence tests is, however, more serious. Berry (2), Brown (4), Jordan (19), Darsie (46) and others have shown that children from foreign homes rank, on the average, somewhat lower in tests requiring the use of English than they rank on non-language scales. On the other hand, the use of a non-language scale does not result in equal ratings for the various nationality-groups. The Italian continues to rank low even on the non-verbal tests; while the average Japanese child who has had three or four years' training in American schools makes a good showing, despite the handicap of language, on such a test as the Stanford-Binet.

In this connection it is interesting to compare races with respect to the persistence of the use of the foreign language in the home. Jordan (19) has collected information on this head from his study of the school children of Minneapolis and St. Paul. His method of treating the data is unfortunately

misleading, since he bases his figures upon the total number of parents of a given nationality, regardless of the length of time they have been in this country. He shows, for each of the eleven nationalities which he considers, the percentage of the total number of parents who have adopted English as the language of preferential use in the home; also the percentages who continue to use the foreign language after ten, twenty, and thirty years' residence in this country. It is not possible, from the data which he gives, to arrive at the really significant figures which would be either the *average length of residence before English is adopted as the home language*, or *the percentage of those who have been in this country ten, twenty, or thirty years and still use the foreign language in the home.*

When the ratio of the number of parents who have been in this country twenty years or longer and still use the foreign language in the home to the total who have either been in this country twenty or more years or (disregarding length of residence) have adopted English as the home language (Jordan, pp. 28–29, Tables 8–9) is calculated, results are obtained as shown in the following table. The median IQs for nine of these nationality-groups as reported by Brown (4) or Murdock (24) are given for comparison. Jordan's results do not include the IQs for his groups.

TABLE I

PERSISTENCE OF FOREIGN LANGUAGE COMPARED WITH INTELLIGENCE

Nationality	Foreign Language Ratio	Median IQ
German	20.1 to 100	102.3
Danish	31.6 " 100	.....
Roumanian Jew	39.7 " 100	98.0
Norwegian	49.2 " 100	103.8
Swedish	53.1 " 100	101.9
Bohemian	76.9 " 100	.....
Austrian	77.8 " 100	99.5
Russian Jew	79.7 " 100	98.0
Italian	86.4 " 100	77.5
Slovak	88.4 " 100	85.6
Finnish	97.4 " 100	90.0

The rank-order correlation between foreign language ratio and IQ, as given above, is − .754. This might be considered

evidence that the use of a foreign language in the home is one of the chief factors in producing mental retardation as measured by intelligence tests. A more probable explanation is that those nationality-groups whose average intellectual ability is inferior do not readily learn the new language.

The results about to be presented are based upon results obtained from 2,457 public school children, practically all of whom were American-born but in whose immediate ancestry a number of racial stocks are represented. Children of mixed parentage have not been included except in the case of the negroes, where no attempt has been made to differentiate between the mulatto and the pure negro, and the Indians, many of whom are "half-breeds." The test used was the Goodenough intelligence test for young children.[1] This test is based upon drawings of the human figure, and is completely independent of language. It would seem, therefore, that it should be entirely fair to the child of foreign parentage. The distribution of intelligence quotients earned on this test by children of different racial origin is shown in Table II, the data being collected from the following schools.

1. Three schools for colored children; one in Chattanooga, Tennessee, one in Mt. Pleasant, Tennessee, and one in Natchitoches, Louisiana.

2. Three schools for white children from the same cities.

3. Three schools from Fresno, California. The enrollment of these schools was largely foreign.

4. One school from Los Angeles, California. This school was made up almost entirely of Italian and Mexican children.

5. The Hoopa Valley Indian School, California.

6. Five kindergartens from Santa Clara County, California.

7. One school from San Jose, California. The population was largely foreign, embracing a number of nationalities.

Grades one to four were included in the case of groups 1, 2, and 5. In the remaining cases only the first three grades were taken. All children enrolled in these grades for whom facts as to racial stock were available are included in these distributions.

Insofar as this scale can be considered a measure of intellectual development, the results are significant. Although the test is entirely independent of language, the South

[1] F. L. Goodenough, *The measurement of intelligence by drawings*, World Book Co., Yonkers, 1926; A new approach to the measurement of the intelligence of young children, *Ped. Sem. & J. Genet. Psychol.*, June, 1926.

TABLE II

DISTRIBUTION OF INTELLIGENCE QUOTIENTS BY RACIAL STOCK

IQ	American	Armenian	Italian	Spanish-Mexican	California Negroes	Southern Negroes	Hoopa Valley Indians	Jewish	Chinese	Japanese	Germans	Portuguese	English and Scotch	French and Swiss	Danish, Swedish and Norwegian	Assyrian, Slavonian and Serbian
160	1		1	1												
150	2		0	0	1											
140	11	1	1	0	0	1		1	1	1	1				1	
130	22	1	3	4	0	1		4	1	4	1	1	1		1	1
120	44	2	15	12	3	8	2	5	4	1	5	0	0	2	3	0
110	75	13	27	27	5	19	1	14	2	7	1	0	4	2	8	5
100	103	20	58	45	4	52	8	11	7	8	6	2	2	1	5	5
90	116	32	100	73	10	84	20	12	4	10	7	4	3	3	5	7
80	69	29	115	93	17	104	22	5	4	8	4	3	2	3	6	3
70	44	16	98	65	17	125	15	3	2	2	3	0	2	2	1	4
60	11	9	34	35	10	155	10			1	1	1		0	1	3
50	2		4	9	2	51	1							1		1
40				3		11										
30						2										
Total cases	500	123	456	367	69	613	79	55	25	42	29	11	14	14	31	29
Mdn.	100.3	91.8	87.5	87.2	82.7	76.5	85.6	106.3	103.1	99.5	98,8	93.3	99.5	92.8	104.5	94.5
Mean	101.5	92.3	89.1	88.5	85.8	78.7	85.6	106.1	104.1	101.9	101.1	94.5	100.2	94.5	103.5	92.8
S.D.	18.3	15.6	16.0	17.5	18.7	17.5	14.1	16.2	18.0	18.0	19.3	16.5	16.8	19.6	17.8	18.8
Coeff. of var.	18.0	16.9	18.0	19.8	21.8	22.2	16.5	15.3	17.2	17.7	19.1	17.5	16.8	20.7	17.2	20.3

European and negro groups rank very much below the American children and those of North European stock. The rank-order of the various nationality-groups corresponds very closely to that found by means of other intelligence tests. (See the list above.) The coefficient of variability is highest for the negroes and lowest for the Jewish children. The high variability of the negroes is probably due to the mulattoes. The comparative homogeneity of the Jewish group may be due either to the effects of selective immigration (these are all California cases) or to the alleged ethnological "purity" of the Jewish race.

There is no reason for thinking these children to be other than fairly representative of their several racial groups as found in this country, except in the case of the American children. In order to be absolutely fair to the foreign groups, it was decided not to include any schools from superior residential districts in these distributions. The population of such schools is, as a rule, made up almost entirely of American children whose intelligence is, on the average, distinctly above that of the American children residing in foreign sections, who make up the bulk of the cases included under this group in Table II.

Summary

1. Children of different racial groups have been found to differ greatly in their performance on a primary group-test designed to measure intellectual capacity. Since the test used is entirely non-verbal, these differences cannot be explained on the basis of a linguistic handicap.

2. Although the test is completely independent of language, the rank-orders of the various racial groups correspond very closely to the results of other investigators using verbal tests.

References

1. Arlitt, A. H. On the need for caution in establishing race norms, *J. Appl. Psychol.*, 1921, 5, 179–183.
2. Berry, C. B. The classification by tests of intelligence of ten thousand first grade children, *J. Educ. Res.*, 1922, 6, 185–203.
3. Brigham, C. A study of American intelligence, *Princeton Univ. Press*, 1923.
4. Brown, G. Intelligence as related to nationality, *J. Educ. Res.*, 1922, 6, 324–327.
5. Bruner, F. C. Racial differences, *Psychol. Bull.*, 1914, 11, 384–386.

6. Cooley, C. H. Genius, fame, and the comparison of races, *Ann. Amer. Acad. Pol. & Soc. Sci.*, 1897, **9**, 317–358.
7. Davenport, C. B. Comparative social traits of various races, *School & Soc.*, 1921, **14**, 344–348.
8. Derrick, S. M. A comparative study of the intelligence of 75 white and 55 colored college students by the Stanford revision of the Binet-Simon scale, *J. Appl. Psychol.*, 1920, **4**, 316–329.
9. Dickson, V. E. *The relation of mental testing to school administration, with special reference to children entering school*, Master's Thesis, Stanford University.
10. Faris, E. The mental capacity of savages, *Amer. J. Soc.*, 1918, **23**, 603–619.
11. Ferguson, G. The psychology of the negro, *Arch. of Psychol.*, 1916, no. 36, 138.
12. Garth, T. R. The results of some tests on full and mixed blood indians, *J. Appl. Psychol.*, 1921, **5**, 359–372.
13. Garth, T. R. A comparison of mental abilities of mixed and full blood indians on a basis of education, *Psychol. Rev.*, 1922, **29**, 221–236.
14. Garth, T. R. White, Indian, and Negro work curves, *J. Appl. Psychol.*, 1921, **5**, 14–25.
15. Garth, T. R. Racial differences in mental fatigue, *J. Appl. Psychol.*, 1920, **4**, 235–244.
16. Herrick, D. S. A comparison of Brahman and Panchama children in South India with each other and with American children by means of the Goddard form board, *J. Appl. Psychol.*, 1921, **5**, 253–260.
17. Hunter, W. S., & Sommermier, E. The relation of the degree of Indian blood to score on the Otis intelligence test, *J. Comp. Psychol.*, 1922, **2**, 257–277.
18. Jordan, A. M. Notes on racial differences, *School & Soc.*, 1922, **16**, 503–504.
19. Jordan, R. H. *Nationality and school progress; a study in americanization*, Pub. School Pub. Co., 1921.
20. McDougall, W. *Is America safe for democracy?* 1921.
21. Mayo, M. J. The mental capacity of the American negro, *Arch. of Psychol.*, 1913, no. 28.
22. Mitchell, I., Rosanoff, I. & A. A study of association in negro children, *Psychol. Rev.*, 1919, **26**, 354–359.
23. Morse, J. A comparative study of white and colored children by the Binet tests, *J. Educ. Psychol.*, 1913, 366–367.
24. Murdoch, K. A study of race differences in New York City, *School & Soc.*, 1920, **11**, 147–150.
25. Myers, C. S. *International race problems*, (G. Spiller, ed.), 1911.
26. Perring, L. F. A study of the comparative retardation of negro and white children in Philadelphia, *Psychol. Clinic*, 1915, **8**, 87–93.
27. Peterson, Joseph. The comparative abilities of white and negro children, *Comp. Psychol. Monog.*, 1923, **1**, (no. 5).
28. Pintner, R., & Keller, R. Intelligence tests of foreign children, *J. Educ. Psychol.*, 1922, **6**, 214–222.
29. Pyle, W. H. A study of the mental and physical characteristics of the Chinese, *School & Soc.*, 1918, **8**, 264–269.
30. Pyle, W. H. The mind of the negro child, *School & Soc.*, 1915, **1**, 357–360.
31. Reuter, E. B. *The Mulatto in the United States*, Badger, 1918.
32. Rowe, E. C. Five hundred forty-seven whites and two hundred sixty-eight Indian children tested by the Binet-Simon scale, *Ped. Sem.*, 1914, **21**, 454–468.

33. SCHWEGLER, R. A., & WINN, E. A comparative study of the intelligence of white and colored children, *J. Educ. Res.*, 1920, **2**, 838–848.
34. SHELDON, W. H. The intelligence of Mexican children, *School & Soc.*, 1924, **19**, 139–142.
35. SUNNE, D. A comparative study of white and negro children, *J. Appl. Psychol.*, 1917, **1**, 71–83.
36. SUNNE, D. Comparison of white and negro children in verbal and non-verbal tests, *School & Soc.*, 1924, **19**, 469–472.
37. STRONG, A. C. Three hundred fifty white and colored children measured by the Binet-Simon scale of intelligence; a comparative study, *Ped. Sem.*, 1913, **20**, 485–515.
38. SYMONDS, P. M. The intelligence of Chinese in Hawaii, *School & Soc.*, 1924, **19**, 442.
39. THOMAS, W. I. Race psychology, *Amer. J. Soc.*, 1912, **17**, 725–775.
40. THORNDIKE, E. L. Intelligence test scores of colored pupils in high schools, *School & Soc.*, 1923, **18**, 569.
41. WALCOTT, G. D. The intelligence of Chinese students, *School & Soc.*, 1920, **11**, 474–480.
42. WOODS, F. A. The racial origin of successful Americans, *Pop. Sci. Mo.*, 1914, **84**, 397–402.
43. WOODWORTH, R. S. Racial differences in mental traits, *Science*, 1910, **31**, 171–186.
44. YOUNG, KWOK T. The intelligence of Chinese children in San Francisco and vicinity, *J. Appl. Psychol.*, 1921, **5**, 267–274.
45. YOUNG, K. Mental differences in certain immigrant groups, *Univ. of Oregon Pub.* 1922, **1** (no. 11).
46. DARSIE, M. L. The mental capacity of American born Japanese children. *Comp. Psychol. Monog.*, 1925.

ANOSMIA AND ITS EFFECTS UPON TASTE PERCEPTIONS

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Injuries to the nasal membranes, or injuries involving the axones of the olfactory receptors in regions superior to these membranes, very frequently prove fatal to smell. The high frequency of this fatality is believed by Mercier-Bellevue (16) to arise from the fact that only one neurone, the receptor itself, leads from the lining of the nostrils to the olfactory bulb and that the cell-body of this neurone is imbedded in this lining exposing it unduly to fatal injury or to pathological involvement. Gilliland has described in this JOURNAL (1921, 4, 318–326) the taste sensitivity of an anosmic subject; and Singer (22) has reported a case of ageusia and anosmia resulting from a probable lesion of the cerebral cortex due to a fall from an automobile truck, the ageusia slowly disappearing while the anosmia permanently remained but was accompanied by an olfactory hallucination. The literature very probably contains descriptions of a great many cases, although the writers have chanced upon only these two instances of total anosmia. The senior writer, however, knows personally of four cases of anosmia, one of which is the subject of the investigation here reported.

The subject, Mr. Y, was a student in the University of Oregon of about twenty years. He was in excellent health and seemed to have no ascertainable physical ailment which could account for his loss of smell. According to his own report, his smell was normal until the summer of 1924. A week or so following an automobile accident (August, 1924) he was made aware rather suddenly of his inability to smell. Whether this loss of sensitivity was incurred by a physical injury, or was due to shock, could not be learned (X-ray photographs of his skull were not taken); although he had

learned of no injury which he may have sustained in the accident. He had not been ill previous to the accident, nor had he been engaged in any occupation calculated to injure his olfactory organs. Although at the time of our investigation he was quite certain of his inability to smell, he was just as certain that he could command olfactory images of smells which had antedated the accident.

The facts suggested the following inquiries; Was Y's anosmia complete? If smell was entirely gone, was taste sensitivity in any way impaired which might be discoverable by investigation of the thresholds of taste? If taste was not impaired, did it, by a heightened sensitivity, compensate for the loss of smell? Were the oral perceptions, as determined by his ability to judge correctly the nature of food and other substances placed in his mouth, below or above normal? And, finally, what—if any—was the difference between him and normal subjects in the use of secondary criteria, such as pressures, temperatures, pains, and kinæstheses in oral perceptions?

Acting as control subjects, Messrs. Goodman and Hockett, experimenting one upon the other, blindfolded themselves and without knowledge determined that they could easily and intensely smell the olfactory substances presently to be enumerated. When a small uncorked vial was placed in the nasal opening (the neck of the vial just filling the orifice) the subject reported whether or not he experienced the odor. These control subjects reported that they smelt bromine, ammonium hydroxide, asafetida, oil of rose, oil of peppermint, camphor, ammonium valerinate, acetic acid, burning sulphur, iodine, oil of wintergreen, each in a concentrated solution; but Subject Y was unable to smell these substances even with deep inhalation or by sniffing. As a result of the application of bromine, acetic acid, burning sulphur, ammonium hydroxide and iodine, Subject Y experienced a strong burning in the mucus of the nasal cavities and the throat. When the stimuli were oil of wintergreen, oil of peppermint, oil of rose and asafetida, sensations of a tingling quality, somewhat pleasing, were localized in the mucus of the back of the mouth

and the throat. But, in neither of the above cases were olfactory qualities experienced by Y. Such evidence inclines us to the view that this subject's anosmia is extensive if not complete. Winkler once described (24) a case of a lack of smell for benzoin but perfect smell for all other olfactory stimuli; Blakeslee has described (3) cases of anosmia for the odor of *verbena* flowers; and Blumenbach has discussed (4) a case of a person anosmic for *mignonette* only. All of these cases would be classified as partial or selective anosmia, whereas our subject would apparently represent the 'complete' type.

In an effort to exhibit the relationship between the subject's tactile sensitivity in the nose and the controls' smell thresholds, we employed gum-resin asafetida, making aqueous solutions at a temperature of 24° centigrade. These solutions were placed in vials of equal-sized necks, just about fitting the nasal openings, which exposed 77 sq mm of the surface of the liquids, which were of various strengths. The first solution used was of the strength of 1 : 50,000. The vial was inserted into one nostril, the other being held shut with the subject's finger while he alternately inhaled deeply and took short sniffs. Occasionally, as a check, vials containing only pure water were used. Under such conditions, and with this strength of stimulus, Subject Y had no sensory experience whatsoever with respect to the smell stimulus. Aronsohn's suggestion (1) that a period of 3 minutes intervene between tests, to allow for recovery from fatigue or exhaustion, was followed throughout the tests. The concentrations were increased until finally the subject could detect, by tactile or other perceptions, but not by smell, the presence of some sort of 'tickling' substance. The weakest concentration was found to be 1 : 175 at 24° centigrade. Subject Y, however, did not invariably detect the stimulus in the strength of 1 : 175; in fact, the concentration of 1 : 150 had in one case to be heated to 32° before the stimulus was tactually or otherwise sensed. The control subjects, tested by a similar procedure, evinced every low thresholds of an olfactory or, possibly, olfactory-tactual-kinæsthetic order. Control I had a threshold

of 1 : 1,200,000. Control II (though afflicted with a nasal cold at the time) showed a threshold of 1 : 450,000. This series of observations convinced us of the completeness of the subject's anosmia; hence, we did not resort to Woodrow's (25) more complicated technique of regulating the number of molecules per liter of air by controlling the vapor tension nor to the more inexact Zwaardemaker olfactometer.

We next attacked the two problems relating to Subject Y's taste sensitivity, working on the four qualities of saline, sour, sweet, and bitter, ignoring the metallic and alkaline qualities proposed as primary by Wundt (27) and later interposed by Henning in his gustatory tetrahedron (11). Inasmuch as Richards (19) had shown that the taste of sour is generally proportional to the concentration of the hydrogen ions, and Kahlenberg (12½) and Crozier (6) that certain deviations from this rule occur with respect to acetic acid, an inorganic acid (hydrochloric) was selected as the stimulus to be used for sour. Sodium chloride became the stimulus for saline experiences, sucrase (cane sugar) for sweet, and quinine sulphate, a very soluble salt of this bitter alkaloid, for bitter. Parker and Stabler (17) had obtained results (possibly because of the greater solubility of their stimulus, quinine hydrochloride) which do not accord with ours to be presently described. Our substances were dissolved in sterile water in various concentrations, and the various dilutions were placed in small vials properly labelled. Clean glass rods were used to transfer the liquid drops to the special areas of the subject's tongue. The subject was not blindfolded; but not being allowed to see the vial from which the drop was taken, he was kept in ignorance of the nature of the taste stimulus. The special areas which were stimulated were those which have been generally assigned to the four qualities; these areas having been pretty definitely and thoroughly explored by Shore (21), Kiesow (13), and Hänig (9). The experiments began by applying a very weak solution to the appropriate locality of the tongue whereupon the subject formed a judgment as to the taste quality before the tongue was withdrawn into the mouth and brought into contact with the palate. A second

judgment was formulated after the tongue had been drawn in and had engaged itself several times with the palate. Each control subject discovered taste to be more sensitive in the latter event, and this was also true of Subject Y. Luciani (14) has explained this heightened sensitivity by the increased salivation which attends movements of the tongue, etc., but psychologists have regarded it as also being partly produced by other factors. The following table (I) presents the thresholds of the anosmic subject and of the two controls.

TABLE I

THRESHOLDS OF TASTE SENSITIVITY OF AN ANOSMIC SUBJECT, TWO CONTROL SUBJECTS, AND THE SUBJECTS OF BAILEY AND NICHOLS (B & N)

Subject	Sensation	Stimulus and Its Temp.	Stimulus Ratio (Threshold)
Y	Bitter	Quinine Sulphate—16° C.	1 : 140,000
I	"	" " "	1 : 110,000
II	"	" " "	*1 : 20,000
? (B & N)	"	" ? ?	1 : 390,000
Y	Sweet	Sucrase 16° C.	1 : 150
I	"	" "	1 : 180
II	"	" "	*1 : 100
? (B & N)	"	Sugar (?) ?	1 : 199
Y	Sour	HCl 16° C.	1 : 1,750
I	"	" "	1 : 2,000
II	"	" "	*1 : 1,000
? (B & N)	"	Sulphuric Acid ?	1 : 2,080
Y	Saline	NaCl 16° C.	1 : 2,000
I	"	" "	1 : 1,750
II	"	" "	*1 : 1,750
? (B & N)	"	" ?	1 : 2,240

* Subject II possessed a cold infection of the throat.

The foregoing results, while they do not present absolute agreement between the results of Bailey and Nichols (2) as cited by Sanford (20) and our own, still they do indicate no great inferiority of our anosmic subject when compared with our two controls. In fact, he seemed to be more sensitive than they to salt and bitter, while one of them was more sensitive than he to sour and sweet. We are therefore inclined to conclude that (1) our subject's anosmia has not deleteriously affected his taste sensitivity and (2) his slight superiority over both controls with respect to salt and bitter

and over one control with respect to sweet and sour is not sufficiently great to lead us to assert that his loss of smell has been compensated for by an increased acuteness of these four taste qualities. It is, of course, obvious that we had no means of knowing what was the delicacy of his taste-sensitivity prior to his accident.[1]

If our subject does not possess compensating taste sensitivities, there still remains the possibility that he may excel the normal subject or that he may be the equal of the normal subject in perceiving, judging, or recognizing the nature of such taste and smell objects as foods, when placed in the oral cavity. This possibility is not unreasonable, though he may lack smell and even though only strong concentrations of olfactory stimuli in the nasal cavities can arouse tactile experiences, giving him secondary cues characteristic of the act of smelling. The writers know of very little literature concerning perceptual patterns involved in taste experiences. Henning (11), probably more than any other investigator, has brought together facts concerning these perceptual experiences, and he has demonstrated that there are a great many taste and smell illusions which arise from the elimination of vision and from the juxtaposing and reorganizing of the contents of the various patterns. Woods (26) has demonstrated that olfactory recognition passes, as a result of practice, through genetic and developmental stages, from the simple and incomplete, through the rich, complete and well-integrated, and then, by steps, to the telescoped, reduced and vicarious, but no less integrated, form. Kinæstheses, derived from sniffing at unfamiliar olfactory objects, and the burning sensations, were shown by her to be important in recognition and themselves undergoing alteration consequent upon practice, these alterations appearing in the various stages of

[1] The old theory of sensory compensation, involving in no way the concept of perceptual accretions arising in practice from secondary criteria after some other sense has been lost or greatly impaired, reminds one of the subtle comment of a man of the street who, when asked if he believed this theory, replied, "Sure! If a man's right leg is shorter than his left, his left is longer than his right!" Seashore and Ling (20½), we believe, gave a fair answer to the theory as it related to the tactile sensitivity of blind and seeing persons; but Brown and Stratton have raised the issue again in this JOURNAL (1925, 8, 434-442).

recognition. On the other hand, Gilliland (8) reached the remarkable conclusion that the anosmic subject "has no greater ability in detecting the complex tastes than the normal subject when using the sense of taste alone"; that is to say, when denied smell and vision but using taste augmented by such secondary qualities as strain, pressures and temperatures. It is not safe to adopt the conclusion of Gilliland unless one can test the perceptual experiences of a blindfolded anosmic subject shortly after the onset of his anosmia and also again many months or years later. One would imagine that with the anosmic person, as with the blind, long years of practice and long years of dependence upon secondary cues, with the consequent redintegrations of taste, would lead to an increased ability to judge correctly the nature of oral stimuli. Boring and others have thrown considerable light upon the experiential patterns of a perceptual order in a subject whose alimentary tract is stimulated here or there by various stimuli.

In an effort to ascertain whether or not Subject Y's gustatory (or oral?) perceptions differed from the normal control's perceptions, we blindfolded him and administered food particles to his tongue. He was to formulate two judgments; one during the extension of the tongue from the mouth with the food particles still in place, this position sometimes being fixed and stationary and at other times being changed by moving the object over the tongue; the other judgment was to be formed after the tongue had been returned into the mouth cavity and after lingual movements and salivation had occurred. We made efforts to change the food stimuli from their usual sizes and shapes, by dicing, mashing, etc. The mouth was thoroughly rinsed with distilled water of a constant temperature after each pair of judgments had been given. These judgments were expressed to the experimenter immediately after the second was formulated by the subject.

The controls submitted to the test before it was tried on the anosmic subject. In their case a blindfold was used, the nostrils were plugged with cotton, and instructions were given

TABLE II

SUBJECTS, ORAL STIMULI AND 1ST, 2D, AND 3D JUDGMENTS OF ANOSMIC AND CONTROL SUBJECTS

Subject	Stimulus	1st Judgment	2d Judgment	3d Judgment
Y	Tomato	Cold, smooth, tasteless	Soft, cold, pretty sour, = orange (?)	
I	Tomato	Smooth, cold, tasteless	Fibrous, cold, = banana	Tomato
II	Tomato	Smooth, cold, tasteless	Sweet, fruity, smooth, = (?)	Tomato
Y	Cream (of choc.)	Cold, soft, sweet, sugary (brown sug.)	Sticky, sweet, moist, = candy	
II	Cream (of choc.)	Smooth, sticky, sweet	Sweeter, sticky, cool, = (?) cream (choc.)	Cream (choc.)
Y	Sweet pickle	Cold, fruity, sweet, juicy, smooth	Smooth, fruity, cold, = pickle	
I	Sweet pickle	Sour, burny, fruity	Sour, acid, burning, = (?)	Sweet pickle
II	Sweet pickle	Slightly sour, stinging, no taste	Burny, acid, sweet, = lemon peel	Sweet pickle
Y	Onion	Smooth, hard, bitter	Smooth, hard, burny, = onion (no smell)	
I	Onion	Rough, hard, tasteless	Burny, bitter, hard, cold, = onion	Onion
II	Onion	Hard, cool, no taste	Fibrous, stingy, burny, = onion	Onion
Y	Banana	Smooth, soft, slightly sweet	Mushy, real sweet, = banana	
I	Banana	Smooth, cold, slightly sweet	Very sweet, sticky, = (?)	Banana
II	Banana	Cold, slightly sweet, smooth	Sweet, smooth, = jam	Banana (?)
Y	Cookie (ginger)	Hard, dry, slightly sweet, rough	Crumbly, sweet, stinging, = ginger cookie	
Y	Cheese, Tillamook	Soft, tasteless, smooth	Sting, no taste, not recognized	
I	Cheese, Swiss	Smooth, cold, burny	Burny, sticky, = cheese (?)	Cheese
II	Cheese, Tillamook	Smooth, cool, tasteless	Bitter, stinging, = cheese (?)	Cheese
Y	Banana and wienie	Tasteless, slick, cold, smooth	Sweet and meaty, wienie and banana	
II	Banana and wienie	Smooth, sweet, cool	Smooth, fibrous, cool, = (?)	(?)
Y	Date	Sweet, sticky, cold, smooth, slick	Sweet, cold, smooth, = date	
I	Date	Rough, tasteless, granular	Sweet, granular, = (?)	Date
Y	Lemon peel	Sour, hard, cool (?)	Sour, rough, cold, = lemon peel	
Y	Wienie	Cold, soft, slightly meaty	Soft, cold, smooth, spicy, = bologna	
I	Chopped peanut	Rough, hard, tasteless	Rough, hard, bitter, = peanut (?)	Peanut

Note: In the above table "?" represents doubt expressed by the subject. In a few cases substances were missed with one or both controls. The serial order of words represents the order of the subject's judgments.

that the breath was to be held preceding and during the two judgments to prevent the passage of particles by air to the nasal epithelium by way of the *choanæ*. Later (with the controls still blinded) the cotton plugging was removed and the experiments were repeated; here the subject was allowed to breathe normally.

The results of these 'recognition' experiments are presented in Table II.

Examination of the foregoing table reveals that, with the possible exception of two or three judgments, Subject Y correctly named the oral stimulus on each second judgment, while neither of the controls could name it correctly with the frequency of Y until the third judgment, which was based upon both mastication, with its attendant cues, and smell. Subject Y's nostrils were unclosed during his second judgments so that, in this respect, these judgments were comparable to the third judgments of the control subjects. His anosmic condition, however, and the correctness of his second judgments, as contrasted with the doubts expressed by the controls on some of their third judgments, would rate Y's second judgments as being superior to those of the controls when they had the additional aid of smell. At any rate, the results seem to demonstrate that, in spite of his anosmia, he is as efficient as normal persons who cannot see, in naming correctly the substance or object placed in his mouth and that he is superior to the blindfolded person who is deprived of smell. The table also suggests that secondary cues may have been of great significance not only to the anosmic

[2] That kinæstheses and other syntheses do play rôles in what are regarded as comparatively simple perceptions has been demonstrated by S. W. Fernberger, An introspective analysis of the process of comparing, *Psychol. Monog.*, 1919, **26**; J. Philippe & J. Claviere, *Rev. phil.*, 1895, **40**, 672–682; H. N. Loomis, *Psychol. Monog.*, 1907, **8**, 334–348; L. B. Hoisington, *Amer. J. Psychol.*, 1920, **31**, 114–146; E. Shults, *Amer. J. Psychol.*, 1922, **33**, 135–139; A. S. Baker, *Amer. J. Psychol.*, 1922, **33**, 139–144; Mueller & Schumann, *Pflüger's Arch.*, 1889, **45**; Goldscheider (see Ortmann, *J. Comp. Psychol.*, 1923, **3**, 12–14); Woodworth, *ibid.*, p. 15; Fechner & Steffens, *Zsch. f. Psychol.*, 1900, **23**; C. N. Waterman, *J. Exp. Psychol.*, 1917, **2**, 289–294; and syntheses, possibly not involving kinæstheses, in comparatively simple perceptions have been demonstrated by I. Bershansky, *Amer. J. Psychol.*, 1922, **33**, 584–587; 1923, **34**, 291–295; Cobbey & Sullivan, *ibid.*, 1922, **33**, 121–127; and L. Knight, *ibid.*, 1922, **33**, 587–590.

subject but also to the controls in reaching both judgments one and two and also judgment three. These secondary cues were in the nature of such characteristics as warmth or coldness, roughness or smoothness, hardness or softness, slickness, stinging and burning. Unfortunately our results contain no descriptions of kinæstheses, except perhaps as these are included in the above listed secondary cues,[2] and no suggestions are given as to their importance in acts of recognizing and judging.

As we have already observed, we had no way of knowing what sensitivities and what abilities of oral recognition our anosmic subject possessed before his accident. He may always have excelled the normal person in the ability to judge, without relying on smell, the nature of food placed in the oral cavity. It is an intriguing possibility, however, that, following upon his accident, practice in using, and dependence upon, secondary cues led to improvement so that he became more skillful than the control subjects.

We have arrived, in summary, at the following conclusions.

1. Subject Y is anosmic; and his anosmia is probably complete.

2. His taste sensitivity, considering only the usual four taste qualities, was not impaired by the accident and its consequences.

3. Our experimental results seem to indicate that his gustatory sensibility has not been made more acute by the accident and by subsequent practice.

4. Y's ability to name correctly the object placed in his mouth even though unaided by vision excelled that possessed by two control subjects under similar conditions.

5. Y's skill in naming food stimuli was developed, as we believe, by the circumstances of his being compelled to attend to and to rely upon secondary cues in his experiences of the anosmic year prior to our observations.

BIBLIOGRAPHY

1. ARONSOHN, E. Experimentelle Untersuchungen zur Physiologie des Geruchs, *Du Bois-Reymond's Arch.*, 1886, 321–357.

2. Bailey, E. H. S., & Nichols, E. L. The sense of smell, *Nature*, 1886, **35**, 74–75.
3. Blakeslee, A. F. Unlike reactions of different individuals to fragrance in verbena flowers, *Science*, 1918, **48**, 298–299.
4. Blumenbach. *Luciani's Physiology*, 1917, Vol. 4, 180.
5. Bawden, H. H. A bibliography of the literature on the organ of the sense of smell, *J. Comp. Neur.*, 1901, **11**, i–xl.
6. Crozier, W. J. Sensory activation by acids, *Amer. J. Physiol.*, 1917–1918, **45**, 323–341.
7. Dimmick, F. L. A note on Henning's smell series, *Amer. J. Psychol.*, 1922, **33**, 423–425.
8. Gilliland, A. R. Taste sensitivity in an anosmic subject, *J. Exp. Psychol.*, 1921, **4**, 318–326.
9. Hänig, D. P. Zur Psychophysik des Geschmackssinnes, *Phil. Stud.*, 1901, **17**, 576–623.
10. Haycraft, J. B. The sense of smell. E. A. Schäfer's *Text-Book of Physiology*, 1898, **2**, 1246–1258.
11. Henning, H. Der Geruch. Leipzig, Barth, 1916. (See also reviews of Henning's work by Gamble, E. A. McC., *Amer. J. Psychol.*, 1921, **32**, 290–295; *Psych. Bull.*, 1916, **13**, 135–137; 1922, **19**, 297–306.)
12. Howell, W. H. *Text-book of physiology*, 1920 (7th ed.), 294–305.
12½. Kahlenberg, L. The action of solutions on the sense of taste, *Bull. Univ. Wisconsin, Sci. Ser.*, 1898, **2**, 1–31.
13. Kiesow, F., & Hahn, R. Beobachtungen über die Empfindlichkeit der hinteren Theile des Mundraums für Tast-, Schmerz-, Temperatur- und Geschmacksreize, *Zsch. f. Psych. u. Physiol. d. Sinnesorg.*, 1901, **26**, 383–417.
14. Luciani, L. *Human physiology*, 1917, Vol. 4, 126–190.
15. Macdonald, M. K. Experimental study of Henning's olfactory qualities, *Amer. J. Psychol.*, 1922, **23**, 535–553.
16. Mercier-Bellevue. *Arch. intern. de laryng., ot., rhyn.*, 1912, **33**.
17. Parker, G. H., & Stabler, E. M. On certain distinctions between taste and smell, *Amer. J. Physiol.*, 1913, **32**, 230–240.
18. Parker, G. H. Smell, taste and allied senses in the vertebrates, Philadelphia & London, Lippincott, 1922.
19. Richards, T. W. Relation of the taste of acids to their degree of dissociation, *Amer. Chem. J.*, 1898, **20**, 121–126.
20. Sanford, E. C. A course in experimental psychology, Boston, Heath, 1908, 47–51.
20½. Seashore, C. E., & Ling, T. L. The comparative sensitiveness of blind and seeing persons, *Psychol. Monog.*, 1918, no. 108, 148–158.
21. Shore, L. E. A contribution to our knowledge of taste sensations, *J. Physiol.*, 1892, **13**, 191–217.
22. Singer, H. D. An unusual syndrome; anosmia and ageusia, *Arch. Neur. & Psychiat.*, 1923, **9**, 262–263.
23. Titchener, E. B. Compensation of odors, *Amer. J. Psychol.*, 1916, **27**, 435–436.
24. Winkler. *Luciani's Physiology*, 1917, **4**, p. 181.
25. Woodrow, H., & Karpman, B. A new olfactometric technique and some results, *J. Exp. Psychol.*, 1917, **2**, 431–447.
26. Woods, E. L. An experimental analysis of the process of recognizing, *Amer. J. Psychol.*, 1915, **26**, 313–387.
27. Wundt, W. *Grundzüge der physiologischen Psychologie*, 1887 (*Aufl.* 3), *Bd.* 1, 411.

A CURVE OF GROWTH DESIGNED TO REPRESENT THE LEARNING PROCESS

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Thurstone [1] has recently determined an empirical equation for the learning process [2] in the form of an hyperbola,[3]

$$Y = \frac{LX}{X + R}, \tag{1}$$

where Y = *attainment* in terms of the number of successful acts performed per unit period,

X = *amount of practice* in terms of total number of practice acts,

L = *limit of attainment* in terms of the number of successful acts performed per unit period,

and R = *ratio* of $L - Y$ to the rate of change of Y with regard to X.

To evaluate R and L, Thurstone "rectifies" equation (1) as

$$L\left(\frac{X}{Y}\right) = X + R. \tag{2}$$

If X/Y is plotted against X, the graph of (2) is a straight line. The straight line which yields the "best" fit for the experimental data is determined by (1) inspection, (2) the method of three equidistant points, (3) the theory of least squares, or (4) the method of the regression equation. The latter two methods give approximately the same values for R and L.

[1] L. L. Thurstone, The learning curve equation, *Psychol. Monog.* (*Stud. Psychol. Lab. Univ. of Chicago*), 1919, **26** (no. 114), 1–51.

[2] Suitable restrictions are made to safeguard the validity of the conclusions. The process is typified by typewriter learning (*op. cit.*, introd., pp. 1, 2, 11).

[3] A modified form of the equation is given in which X is replaced by $X + P$, where P represents the actual or equivalent practice previous to the experiment. However, by setting $X' = X + P$ the modified form returns to equation (1), where initially X' has the value P.

Professor D. B. Klein of the University of Texas has raised the question as to the relation between Thurstone's hyperbolic empirical law and an exponential or growth law. Other investigators have suggested the latter form and, indeed, Thurstone discusses in a very limited fashion the exponential equation in the form

$$Y = L\left(1 - \frac{1}{C^{ax}}\right).$$

He states that the value of C and a cannot be found by rectification unless the value of L is assumed or found by other methods.[4] He dismisses from consideration the growth curve of this form with the assertion that this type does not fit the data nearly so well as the hyperbolic equation (1) does.

It is the purpose of this note to exhibit the important and close relationship between Thurstone's empirical result and the growth-curve or exponential law. This dependence assumes special interest when the growth-curve is obtained by using the values of R and L from equation (1). A striking symmetry between the *speed-amount*[5] curve and the *time-time*[6] curve is put into evidence which is not present in the original work and which thereby introduces remarkable simplicity. An indication is given whereby it may be possible by passing from the growth-law itself to the differential equation underlying it to state certain laws of learning (or forgetting).[7]

II

We will assume that the numbers R and L of equation (1) have been found by means of (2). Consider alongside of (1) the equation

$$Y = L(1 - e^{-(X/R)}). \tag{3}$$

The curves representing (1) and (3) are almost identical in shape for positive values of X; as X increases without

[4] For the modified form of the equation (footnote 3, above) it is necessary to assume or find L and P before rectifying.

[5] *Op. cit.*, p. 19.

[6] *Op. cit.*, p. 21.

[7] A discussion of these laws, together with complete mathematical details and graphs for the curves, is to be found in a forthcoming article (The kinetics of learning, *Amer. Math. Mo.*, 1926).

limit, both (2) and (3) yield L as the limiting value of Y; both pass through the origin of the coördinate system.

It may readily be seen after inverting equation (1) that in place of (2),

$$\frac{1}{Y} = \frac{R}{L}\frac{1}{X} + \frac{1}{L} \tag{4}$$

will serve in place of (2) to rectify (1) and solve for R and L. Using these same values for both (1) and (3), the relationship between these equations becomes readily apparent. For (4), which is the equivalent of (1), may be put in the form

$$\frac{L}{Y} = 1 + \frac{R}{X}. \tag{4'}$$

Now, from (3) we have

$$\frac{L}{Y} = \frac{1}{1 - e^{-(X/R)}} \tag{3'}$$

or

$$\frac{L}{Y} = 1 + \frac{R}{X} - \frac{e^{X_1/R}}{2\left(1 + \frac{X}{R}e^{X_1/R}\right)}, \tag{5}$$

where X_1 is a value between the initial value of X and the variable value of X in the equation.[8]

We note that (4') may be regarded as the approximation to the first order of (5), since the right-hand side of (4') is identical with the first two terms on the right of (5). The last term of (5) represents the difference. From a mathematical point of view, *the hyperbolic curve is a first order approximation of the growth-curve.*

Equation (3) is called the *speed-amount* curve. If we set $t = \frac{1}{Y}$, we get the attainment measured in the number of units of time required to perform a unit successful-act. The equation relating t and X is the *time-amount* curve.

[8] This result may be obtained from (3') by the application of integration by parts to approximations (see Porter and Ettlinger, Elementary calculus, The Century Company, New York, 1925, 209–210). The result may also be obtained from the Extended Law of the Mean (Taylor's formula with remainder); see formula (6), *ibid.*, 211).

Thurstone's form obtained from (1) is

$$t = t_L\left(1 + \frac{R}{X}\right), \tag{1'}$$

an hyperbola again. From (4) we obtain the corresponding equation

$$t = \frac{t_L}{1 - e^{-(X/R)}}, \tag{3''}$$

where $t_L = \frac{1}{L}$ = limit of attainment measured in units of time required to perform a unit successful-act. If we interpret t as the rate of change of time of practice, T, taken with respect to the amount of practice, X, we have

$$\frac{dT}{dX} = \frac{t_L}{1 - e^{-(X/R)}}.$$

Integrating with respect to T from 0 to T and for X from P to X, we obtain [9]

From (3'') and (6) we obtain [10]

$$T = \frac{R}{L}\log_e \frac{Lt_L - 1}{Lt - 1}. \tag{7}$$

If we invert (7) by solving for t in terms of T we obtain [11]

$$t = t_L + (t_P - t_L)e^{-(LT/R)}, \tag{8}$$

where t_P is the value of t corresponding to $T = 0$ or $X = P$.

Equations (7) and (8) are the *time-time* curves.

[9] Equation (6) is to be compared with Thurstone's equation (53) (*op. cit.*, 21)

$$T = \frac{X}{L} + \frac{R}{L}\log_e (X + P) + C_1.$$

$$T = \frac{R}{L}\log_e \frac{e^{X/R} - 1}{e^{P/R} - 1}. \tag{6}$$

[10] Equation (7) is to be compared with Thurstone's equation (54) (*op. cit.*, 21)

$$T = \frac{R + P(1 - Lt)}{(Lt - 1)L} + \frac{R}{L}\log_e \frac{R}{Lt - 1} + C_2.$$

[11] Thurstone has no equation analogous to (8), since his equation (54) (see footnote 10, above) does not admit of a solution for t.

By taking the reciprocal of (8) we find the *speed-time* curve,[12]

$$Y = \frac{LY_0}{Y_0 + (L - Y_0)e^{-(LT/R)}}. \tag{9}$$

The curve (9) has a point of inflection when $Y = \frac{L}{2}$. This result differs from Thurstone's, which is $Y = \frac{L}{3}$ as the point of inflection of his equation (55).

III

The numerical values of the data used by Thurstone are not available in his monograph. It would be of considerable interest to examine these and to endeavor to ascertain whether they fit our growth-curve (3) when obtained by regarding the hyperbola as an approximation of the first order. The writer is, however, not unmindful of the fact that it is entirely within the realm of possibility that the experimental error may well be greater than the difference between equations (1) and (3) as given by the last term of (5) above.

If there is any physiological basis for regarding the learning process as a growth, may we not regard equation (3) as the theoretical or rational[13] equation, even though the above verification be impossible. Certainly the hyperbolic equation is purely empirical. The symmetry of form displayed by the equations (6) and (7), (8) and (9), as compared with the absence of this property in those derived directly from (1), would justify the use of (3) from the point of view of simplicity.

Behind the growth-curve there is a differential equation which may furnish us with a set of dynamical laws of learning.

[12] This curve is not obtained by Thurstone. If, however, we replace t by $\frac{1}{Y}$ in (7) above we obtain

$$T = \frac{R}{L}\log_e \frac{Y(L - Y_0)}{Y_0(L - Y)},$$

which, except for his first term, is substantially Thurstone's (55) (*op. cit.*, 22)

$$T = \frac{R}{L - Y} + \frac{R}{L}\log_e \frac{Y}{L - Y} + C_2.$$

[13] *Cf.* Thurstone, *op. cit.*, part I, p. 10.

One such example may be obtained by differentiating the *speed-amount* curve (3). The resulting differential equation is [14]

$$RY' + Y = \mathrm{L}, \tag{10}$$

where Y' is the rate of change of the speed of attainment Y with respect to the amount of practice, X. If we let $y = L - Y$, (10) takes the form

$$\frac{y'}{y} = -\frac{1}{R}. \tag{11}$$

If we define $L - Y$ as the *margin of attainment*, then (11) can be stated as follows:

The relative rate of decrease of the margin of attainment with respect to the amount of practice is constant.

[14] The *difference* equation $R\Delta Y + Y = L$, where ΔY is the *increase* in speed of attainment per practice period, would be a better formulation.